

Aligning the Forces – Eliminating the Misalignments in IMU Arrays

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Abstract—Ultra-low-cost single-chip inertial measurement units (IMUs) combined into IMU arrays are opening up new possibilities for inertial sensing. However, to make these systems practical for researchers, a simple calibration procedure that aligns the sensitivity axes of the sensors in the array is needed. In this letter, we suggest a novel mechanical-rotation-rig-free calibration procedure based on blind system identification and a Platonic solid printable by a contemporary 3D-printer. The IMU array is placed inside the Platonic solid and static measurements are taken with the solid subsequently placed on all sides. The recorded data are then used together with a maximum likelihood based approach to estimate the inter-IMU misalignment and the gain, bias, and sensitivity axis non-orthogonality of the accelerometers. The effectiveness of the method is demonstrated with calibration results from an in-house developed IMU array. Matlab-scripts for the parameter estimation and production files for the calibration device (solid) are provided.

I. INTRODUCTION

THE development of the micro-electrical-mechanical system (MEMS) technology has revolutionized the inertial sensor industry, making it possible to manufacture large volumes of ultra-low-cost inertial sensors for mass market products. Today, one can get a full six degrees-of-freedom IMU at a size of $3 \times 3 \times 1$ mm for a few dollars. Unfortunately, these IMUs still cannot provide the accuracy needed in, for example inertial navigation applications. However, with the size and price of today's ultra-low-cost IMUs, it is now feasible to construct large arrays of IMUs, and fuse the information from several sensor units, to attain performance and price-size-cost figures not previously seen from MEMS IMUs. See [1] for a review of additional merits of multi-IMU systems.

Low-cost IMUs are generally delivered uncalibrated [2]. Further, due to imperfections in the integrated circuit (packaging) and in the fabrication of the IMU array, the sensitivity axes of the IMUs in the array will not be perfectly aligned. Thus, before the information from the IMUs is fused, the individual IMUs should be calibrated, and the inter-IMU misalignment compensated for. Traditional (redundant) IMU calibration requires expensive dedicated mechanical rotation rigs, see e.g., [3,4,5]. Therefore, simplified calibration methods that do not require a rotation rig have been proposed, see e.g., [6,7,8]. These methods exploit the prior knowledge about the magnitude of the gravity vector to do a blind system identification, but are currently limited to single IMU setups. Consequently, in this letter, the maximum likelihood based blind system identification method described in [6] is extended

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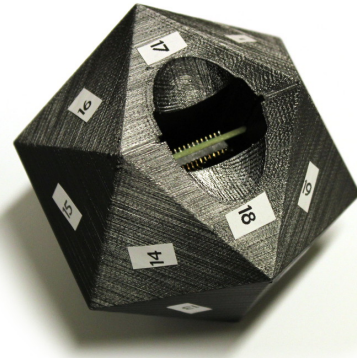


Fig. 1. Icosahedron for IMU array calibration. The IMU array is placed inside the body. By subsequently placing the body on all sides, an even distribution of orientations is provided for the system identification.

to IMU arrays by modeling and estimating the inter-IMU misalignments, in addition to the gain, bias, and sensitivity axis non-orthogonality of the individual IMUs. (We recommend the interested reader to also look at [9], where the calibration of an array of magnetometers is studied; to our knowledge [9] is the only previous example of a similar calibration method applied to a similar array setup.) Further, the accuracy of the estimates is dependent on the excitation of the sensors, i.e., the orientations that the IMU array is placed in during the calibration. Therefore, we propose using an icosahedron (see Fig. 1) to place the array in a set of evenly distributed, but unknown, orientations. The proposed estimation method and the calibration body (printed by a contemporary 3D-printer) is then used to estimate the calibration parameters of an in-house developed IMU array. The results of the calibration show that the sensor misalignment parameters can be estimated consistently, and that the effect of the misalignment compensation is significant.

Reproducible research: SCAD-code and SDL-files for the icosahedron together with a Matlab implementation of the calibration procedure and the data used to produce the results in the paper are provided at www.openshoe.org.

II. PARAMETER ESTIMATION

Taking into account only the most significant error sources, the output $\mathbf{y}_n^{(i)} \in \mathbb{R}^3$ at orientation n of the i :th IMU's accelerometer triad can be described by the model [8,10,11]

$$\mathbf{y}_n^{(i)} = \mathbf{K}^{(i)} \mathbf{L}^{(i)} \mathbf{u}_n^{(i)} + \mathbf{b}^{(i)} + \mathbf{v}_n^{(i)} \quad \begin{array}{l} n = 1, \dots, N \\ i = 1, \dots, M \end{array}$$

where $\mathbf{K}^{(i)} = \text{diag}(\mathbf{k}^{(i)})$ and $\mathbf{L}^{(i)} = \text{unitri}(\mathbf{l}^{(i)})$ are 3×3 diagonal- and unitriangular-matrices, respectively. Fur-

ther, $\mathbf{k}^{(i)} = [k_x^{(i)} k_y^{(i)} k_z^{(i)}]^\top$, $\mathbf{l}^{(i)} = [l_{yz}^{(i)} l_{zx}^{(i)} l_{xy}^{(i)}]^\top$, $\mathbf{b}^i = [b_x^{(i)} b_y^{(i)} b_z^{(i)}]^\top$, and $\mathbf{v}_n^{(i)} \in \mathbb{R}^3$ denote the sensor gain, sensitivity axis non-orthogonality, bias, and noise, respectively. Moreover, $\mathbf{u}_n^{(i)} \in \mathbb{R}^3$ denotes the true force exerted unto the i :th accelerometer triad at orientation n , and N and M denote the number of orientations and IMUs, respectively.

Due to imperfections in the mounting of the IMUs, the coordinate axes of the different IMUs will not be perfectly aligned. To model these misalignments, let $\mathbf{R}_{(i)}^{(j)} \in \mathbb{SO}(3)$ denote the (unknown) rotation matrix that describes the true orientation between the coordinate system instrumented by the i :th IMUs sensitivity axes and the j :th IMUs sensitivity axes. Next, assume that the alignment errors $\xi^{(j)} = [\xi_x^{(j)} \xi_y^{(j)} \xi_z^{(j)}]^\top$ in the mounting of the j :th IMU are small, i.e., below a few degrees. Then, to the first order, the rotation matrix $\mathbf{R}_{(i)}^{(j)}$ can be approximated as $\mathbf{R}_{(i)}^{(j)} = (\mathbf{I} + [\xi^{(j)}]_\times) \bar{\mathbf{R}}_{(i)}^{(j)}$. Here, \mathbf{I} denotes the 3×3 identity matrix and $[\xi^{(i)}]_\times$ denotes the skew-symmetric matrix¹ representation of the alignment errors. The rotation matrix $\bar{\mathbf{R}}_{(i)}^{(j)} \in \mathbb{SO}(3)$ describes the orientation between the i :th and j :th IMU, if they were mounted without any errors; $\bar{\mathbf{R}}_{(i)}^{(j)}$ is assumed known. Thus, the output of the j :th accelerometer triad can, as a function of the input to the i :th accelerometer triad and the parameters $\theta^{(j)}$, be modeled as:

$$\mathbf{y}_n^{(j)} = f(\theta^{(j)}, \mathbf{u}_n^{(i)}) + \mathbf{v}_n^{(j)}$$

where

$$f(\theta^{(j)}, \mathbf{u}_n^{(i)}) = \begin{cases} \mathbf{K}^{(i)} \mathbf{L}^{(i)} \mathbf{u}_n^{(i)} + \mathbf{b}^{(i)}, & j = i \\ \mathbf{K}^{(j)} \mathbf{L}^{(j)} (\mathbf{I} + [\xi^{(j)}]_\times) \bar{\mathbf{R}}_{(i)}^{(j)} \mathbf{u}_n^{(i)} + \mathbf{b}^{(j)}, & j \neq i \end{cases}$$

and where the unknown model (calibration) parameters are

$$\theta^{(j)} = \begin{cases} [\mathbf{k}^{(i)}, \mathbf{b}^{(i)}, \mathbf{l}^{(i)}]^\top, & j = i \\ [\mathbf{k}^{(j)}, \mathbf{b}^{(j)}, \mathbf{l}^{(j)}, \xi^{(j)}]^\top, & j \neq i \end{cases}$$

Note that $\mathbf{K}^{(j)} \mathbf{L}^{(j)} (\mathbf{I} + [\xi^{(j)}]_\times)$ has nine degrees of freedom and consequently a full matrix could have been used instead. However, the suggested parameterization has the advantage that the parameters have natural physical interpretations, and individual parameters can be removed from the calibration.

To estimate the parameters $\theta^{(j)}$, rewrite the input vector in spherical coordinates as $\mathbf{u}_n^{(i)} = \alpha_n \mathbf{s}(\phi_n, \psi_n)$, where $\mathbf{s}(\phi_n, \psi_n) = [-\sin(\phi_n) \cos(\phi_n) \sin(\psi_n) \cos(\phi_n) \cos(\psi_n)]^\top$. Here, ϕ_n and ψ_n denote the i :th IMU's (unknown) pitch and roll, respectively. Now, if the IMU array is stationary, then the magnitude of the input vector $\alpha_n = g$, where g is the magnitude of the local gravity vector. Thus, when the array is stationary, the input vector has only two degrees of freedom, whereas each accelerometer triad provides an estimate of the force vector in \mathbb{R}^3 . This implies that by placing the array in at least twelve (nine) different non-coplanar orientations, the twelve (nine) unknown parameters $\theta^{(j)}$ ($\theta^{(i)}$) of each triad can be estimated.

Assuming the measurement noise $\mathbf{v}_n^{(j)}$ to be white, Gaussian distributed, and uncorrelated between the IMUs, i.e., the covariance matrix $\mathbb{E}\{\mathbf{v}_n^{(i)} (\mathbf{v}_\ell^{(j)})^\top\} = Q_n^{(i)} \delta_{i,j} \delta_{n,\ell}$, where $\mathbb{E}\{\cdot\}$

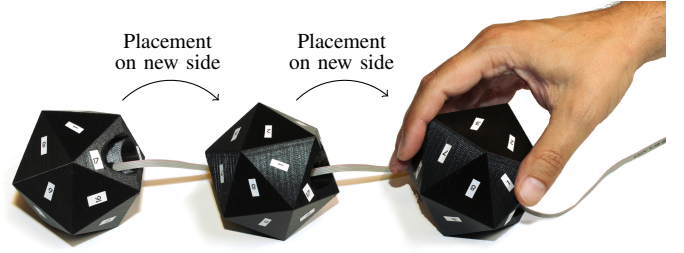


Fig. 2. To acquire calibration measurement data from an even distribution of orientations, the icosahedron with the IMU array is subsequently placed on all sides, and the data are recorded. The numbering on the sides helps with control and the bookkeeping of the orientations.

and $\delta_{i,j}$ denote the expectation operator and the Kronecker delta function, respectively; then the maximum likelihood estimate of the parameters $\{\theta^{(j)}\}_{j=1}^M$ is given by [12]

$$\{\hat{\theta}^{(j)}\}_{j=1}^M = \underset{\{\theta^{(j)}\}_{j=1}^M, \{\phi_n, \psi_n\}_{n=1}^N}{\operatorname{argmin}} \sum_{j=1}^M \sum_{n=1}^N \|\mathbf{y}_n^{(j)} - f(\theta^{(j)}, g\mathbf{s}(\phi_n, \psi_n))\|_{\mathbf{Q}_n^{(j)}}^2$$

where $\|\mathbf{a}\|_{\mathbf{P}}^2 = \mathbf{a}^\top \mathbf{P}^{-1} \mathbf{a}$. The minimization generally needs to be done numerically using, e.g., the Newton-Raphson method. To reduce the risk of the minimization algorithm getting stuck at a local minimum, the initial parameter values should be set to the nominal values given in the data sheet of the IMU. Initial estimates for the pitch and roll may be calculated as $\phi_n^{\text{init}} = \operatorname{atan2}([y_n^{(j)}]_y, [y_n^{(j)}]_z)$ and $\psi_n^{\text{init}} = \operatorname{atan2}(-[y_n^{(j)}]_x, \sqrt{[y_n^{(j)}]_y^2 + [y_n^{(j)}]_z^2})$, respectively. Here $[\mathbf{a}]_k$, $k \in \{x, y, z\}$ denotes the k element of the vector \mathbf{a} .

III. MEASUREMENT METHOD

For the estimation of $\{\theta^{(j)}\}_{j=1}^M$ to be well-conditioned, the orientations $\{\mathbf{s}(\phi_n, \psi_n)\}_{n=1}^N$ should be evenly distributed over the unit sphere. Further, to average out stochastic and unmodeled errors, e.g. non-linearities and cross-axis sensitivities, the orientations should be more than twelve. The number of orientations N used in the calibration is a trade-off between accuracy and execution time. However, the number of desired orientations makes achieving the even distribution a practical obstacle. This can be solved by a simple calibration rig. A Platonic solid provides sides with an even distribution of orientations. If the IMU array is inserted in such a solid, then as illustrated in Fig. 2, an even distribution of orientations is achieved by subsequently placing the polyhedron on all its sides. The Platonic solid with the most sides is the icosahedron. Such a body with an insertion slot for the IMU array, as the one shown in Figs. 1 and 2, can easily be printed with a 3D-printer or ordered from a 3D-printing service. Note that since the orientations of the sides are not assumed known, the requirements on the print quality are modest and imperfections in the print (or incorrect placement on some side) will not have any significant effect on the calibration.

IV. EXPERIMENT

An IMU array has been constructed around 18 MPU-9150 IMUs from Invensense and an AT32UC3C2512 micro-controller from Atmel. See Fig. 3 and [1] for details about

¹The skew-symmetric matrix $[\mathbf{a}]_\times$ is defined so that $[\mathbf{a}]_\times \mathbf{b} = \mathbf{a} \times \mathbf{b}$.

TABLE I
MEAN (RANGE) OF THE PARAMETER ESTIMATES FROM 10 CALIBRATIONS, WITH THE MAXIMUM AND MINIMUM PARAMETER ESTIMATES BELOW.

IMU	k_x [-]	k_y [-]	k_z [-]	b_x [m/s ²]	b_y [m/s ²]	b_z [m/s ²]	l_{yz} [°]	l_{zy} [°]	l_{zx} [°]	ξ_x [°]	ξ_y [°]	ξ_z [°]
1	1.009 (4.7e-4)	1.007 (2.7e-4)	0.994 (1.0e-3)	-0.23 (1.2e-2)	-0.03 (3.9e-3)	-0.94 (1.8e-2)	0.02 (6.1e-2)	0.39 (5.1e-2)	0.08 (1.2e-1)	-0.03 (1.4e-1)	-0.15 (8.1e-2)	-0.66 (2.9e-2)
2	1.002 (5.7e-4)	1.006 (2.6e-4)	0.998 (6.8e-4)	-0.28 (1.5e-2)	-0.05 (2.6e-3)	-0.48 (4.5e-3)	0.01 (7.3e-2)	-0.01 (5.5e-2)	-0.02 (6.2e-2)	0.23 (7.8e-2)	-0.30 (6.3e-2)	-0.16 (2.8e-2)
3	1.003 (9.9e-4)	1.000 (2.3e-4)	1.006 (7.0e-4)	-0.38 (2.1e-2)	0.01 (2.1e-3)	-0.02 (7.4e-3)	-0.00 (9.5e-2)	0.29 (3.8e-2)	-0.04 (7.3e-2)	0.23 (9.7e-2)	0.02 (6.9e-2)	0.61 (2.6e-2)
4	0.997 (5.5e-4)	1.006 (2.4e-4)	0.995 (1.4e-3)	-0.33 (1.9e-2)	0.03 (2.0e-3)	-1.37 (2.2e-2)	-0.01 (7.0e-2)	0.16 (7.0e-2)	0.10 (1.5e-1)	0.08 (1.7e-1)	-0.27 (1.1e-1)	-1.68 (2.6e-2)
5	1.011 (7.3e-4)	0.996 (2.2e-4)	1.011 (1.3e-3)	-0.26 (1.6e-2)	-0.03 (2.3e-3)	-0.98 (1.1e-2)	-0.01 (6.7e-2)	-0.20 (9.7e-2)	-0.27 (7.8e-2)	0.40 (8.1e-2)	-0.37 (8.9e-2)	0.72 (2.7e-2)
6	1.003 (9.0e-4)	1.006 (1.8e-4)	1.006 (6.3e-4)	-0.37 (1.6e-2)	-0.06 (3.8e-3)	-1.03 (2.6e-2)	-0.01 (8.0e-2)	0.02 (8.0e-2)	0.31 (7.9e-2)	-0.12 (9.1e-2)	-0.36 (3.8e-2)	0.57 (2.7e-2)
7	0.999 (5.7e-4)	0.999 (2.1e-4)	0.997 (9.6e-4)	-0.25 (1.3e-2)	-0.03 (5.1e-3)	-1.63 (1.9e-2)	-0.03 (5.7e-2)	0.08 (6.4e-2)	-0.13 (1.1e-1)	0.28 (1.2e-1)	-0.35 (7.8e-2)	-0.04 (2.7e-2)
8	1.008 (6.2e-4)	1.007 (2.1e-4)	0.992 (1.5e-3)	-0.38 (1.5e-2)	-0.03 (2.0e-3)	-1.38 (8.5e-3)	0.01 (6.9e-2)	-0.28 (4.8e-2)	0.06 (1.3e-1)	-0.05 (1.6e-1)	-0.26 (7.6e-2)	-0.50 (2.6e-2)
9	0.998 (1.6e-4)	0.999 (4.8e-4)	1.009 (5.2e-4)	0.18 (3.9e-3)	0.15 (1.1e-2)	0.66 (2.1e-2)	0.01 (4.1e-2)	0.19 (5.1e-2)	0.37 (5.4e-2)	0.41 (6.0e-2)	-0.07 (6.0e-2)	-0.06 (3.6e-2)
10	1.010 (1.3e-4)	0.999 (4.9e-4)	1.015 (5.6e-4)	0.11 (3.2e-3)	-0.07 (9.2e-3)	-1.36 (1.1e-2)	0.04 (3.8e-2)	0.21 (4.0e-2)	-0.08 (2.7e-2)	0.46 (6.0e-2)	-0.38 (8.0e-2)	-0.25 (2.5e-2)
11	1.001 (1.3e-4)	1.008 (4.1e-4)	1.007 (6.3e-4)	0.15 (2.5e-3)	0.13 (1.3e-2)	0.44 (2.0e-2)	0.01 (5.1e-2)	0.11 (5.7e-2)	0.39 (6.3e-2)	-0.33 (6.4e-2)	0.17 (5.5e-2)	-0.38 (3.9e-2)
12	1.007 (1.2e-4)	1.010 (3.8e-4)	1.013 (3.1e-4)	0.15 (3.8e-3)	0.01 (9.3e-3)	-1.77 (9.3e-3)	-0.01 (4.1e-2)	0.31 (3.6e-2)	0.05 (2.4e-2)	-0.57 (5.7e-2)	0.23 (3.1e-2)	-0.54 (3.6e-2)
13	0.999 (2.2e-4)	1.002 (4.8e-4)	0.996 (5.1e-4)	0.16 (5.5e-3)	0.11 (9.5e-3)	0.87 (1.8e-2)	-0.01 (3.9e-2)	-0.24 (5.3e-2)	0.09 (3.5e-2)	0.18 (3.7e-2)	0.79 (2.5e-2)	0.38 (1.4e-2)
14	1.007 (2.2e-4)	1.002 (3.3e-4)	1.002 (6.0e-4)	0.10 (3.3e-3)	0.07 (8.9e-3)	-0.45 (1.7e-2)	-0.00 (5.1e-2)	-0.16 (5.6e-2)	0.08 (4.4e-2)	-	-	-
max	1.011	1.010	1.015	0.176	0.146	0.865	0.042	0.389	0.390	0.465	0.788	0.722
min	0.997	0.996	0.992	-0.380	-0.074	-1.766	-0.033	-0.281	-0.273	-0.566	-0.375	-1.682

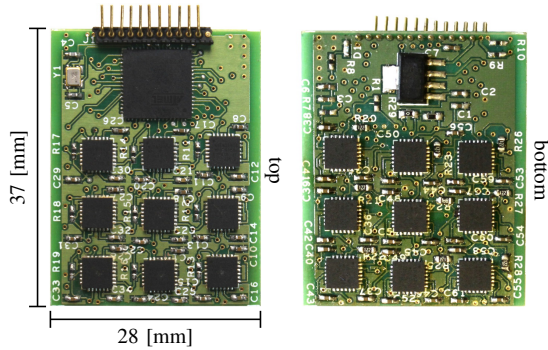


Fig. 3. The in-house constructed IMU array platform holding 18 MPU9150 IMUs (9 on the top side and 9 on the bottom side) and an AT32UC3C2512 microcontroller (top side). The platform is displayed in actual size.

the array. To verify the effectiveness of the suggested calibration procedure, 10 calibration sets, each with the icosahedron placed on all 20 sides, were recorded. The maximum likelihood estimates of the gain, bias, sensitivity axis non-orthogonality, and misalignment of the IMUs in the array were then calculated. The mean and range of the estimates $\{\hat{\theta}^{(j)}\}_{j=1}^M$ from the 10 calibrations are shown in Table I.² Note that the coordinate axes of IMU number 14 was set to define the reference coordinate system of the array, i.e., $i = 14$, and thus no alignment errors were estimated for this IMU. The spread of the mean values in Table I, compared to the range values, shows that the 10 calibrations are consistent.

In the end, compensating for the misalignment calibration should produce more consistent measurements from the IMU array. A natural figure of merit is the spread between the measurements in terms of the sample covariance matrix $\text{cov}(\{\hat{\mathbf{u}}_n^{(i)}; \bar{\theta}^{(i)}\}_{i=1}^M)$ where $\{\hat{\mathbf{u}}_n^{(i)}; \bar{\theta}^{(i)}\}_{i=1}^M$ denotes the compensated forces $\{\hat{\mathbf{u}}_n^{(i)}\}_{i=1}^M$ measured by the IMUs, given the mean parameters in Table I $\{\bar{\theta}^{(i)}\}_{i=1}^M$. Since the covariance depends on the orientation, we average it over all 20 orientations. Then, the improved consistency can be quantified by the ratio

$$\frac{\frac{1}{N} \sum_{n=1}^N \text{tr}(\text{cov}(\{\hat{\mathbf{u}}_n^{(i)}; \bar{\theta}^{(i)}\}_{i=1}^M))}{\frac{1}{N} \sum_{n=1}^N \text{tr}(\text{cov}(\{\hat{\mathbf{u}}_n^{(i)}; \bar{\theta}_{\text{red}}^{(i)}\}_{i=1}^M))} = \frac{(0.0077)^2 [(m/s^2)^2]}{(0.11)^2 [(m/s^2)^2]} \approx -23 \text{ [dB]}$$

where the calibration values $\{\bar{\theta}_{\text{red}}^{(j)}\}_{j=1}^M$ are the results from calibrating the gain, bias, and non-orthogonality of each IMU individually (no misalignment), as originally suggested in [6].

²Due to problems in the printed circuit board assembling process, only 14 out of the 18 IMUs in the array worked as intended during the calibration.

V. DISCUSSION AND CONCLUSIONS

We have suggested a simple calibration procedure for IMU arrays, only dependent on a simple calibration device printable by a contemporary 3D printer. The calibration procedure has been shown to give consistent results for an in-house developed IMU array. Finally, the effectiveness and significance of the misalignment compensation have been demonstrated in terms of a substantially improved consistency (-23[dB]) of force measurements from different IMUs. The magnitude of the alignment errors are such that the resulting errors are comparable or larger than those of individual IMUs and consequently have to be compensated for in order to make a sensible fusion of data. In summary, calibration and misalignment compensation of low-cost IMU arrays are necessary and the simplicity of the procedure helps making such systems practical and accessible for researchers and system developers.

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