Fusing The Information From Two Navigation Systems Using An Upper Bound On Their Maximum Spatial Separation

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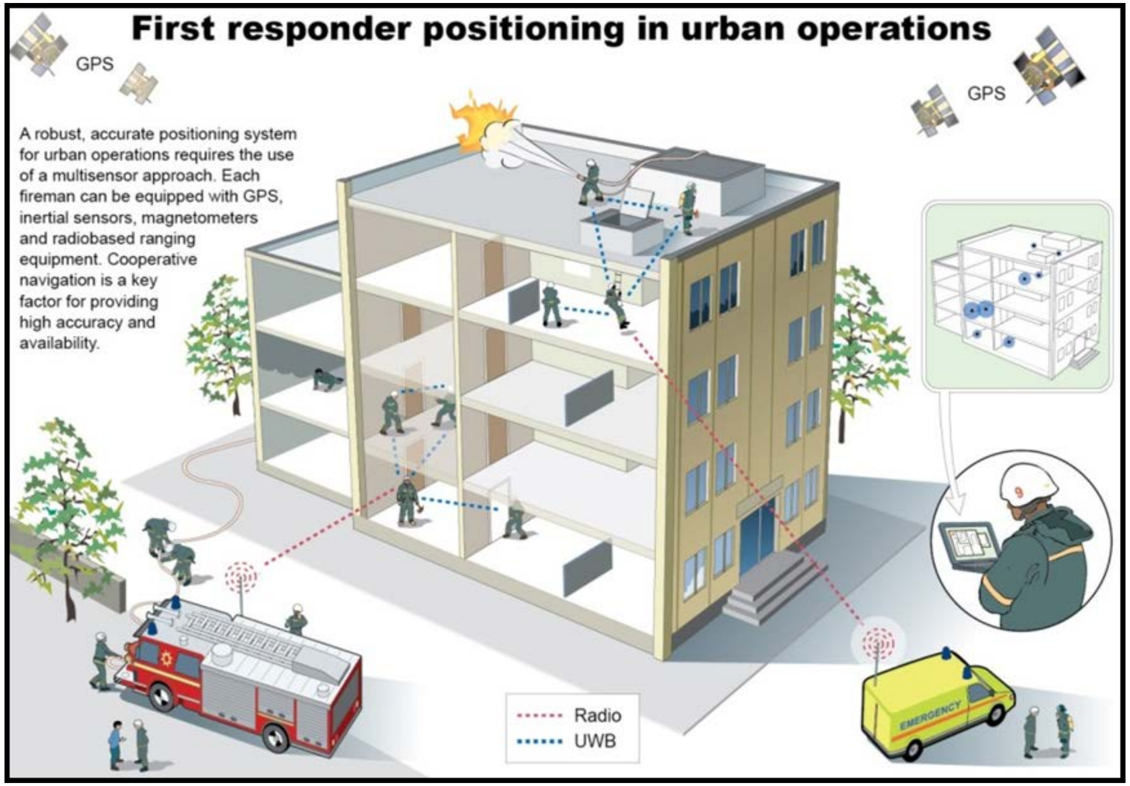
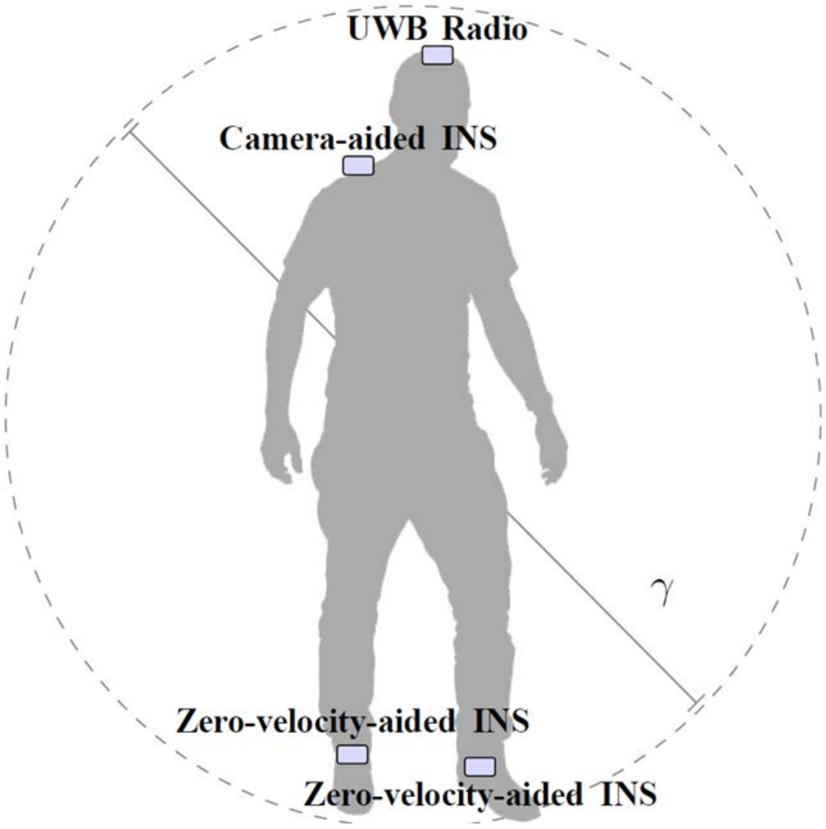
Background

Currently, there is no navigation technology that, on its own, can provide a reliable, robust, and infrastructure-free solution to the problem of positioning a pedestrian in all kinds of indoor environments.



Problem description

- Navigation technologies with complementary properties have different ”optimal” positions on the body.
- The different systems tracks the states of different points on the body.
- There is a non-rigid realtionship between the navigation points.
- There is an upper limit γ how spatially separated the systems can be.



Mathematical problem formulation

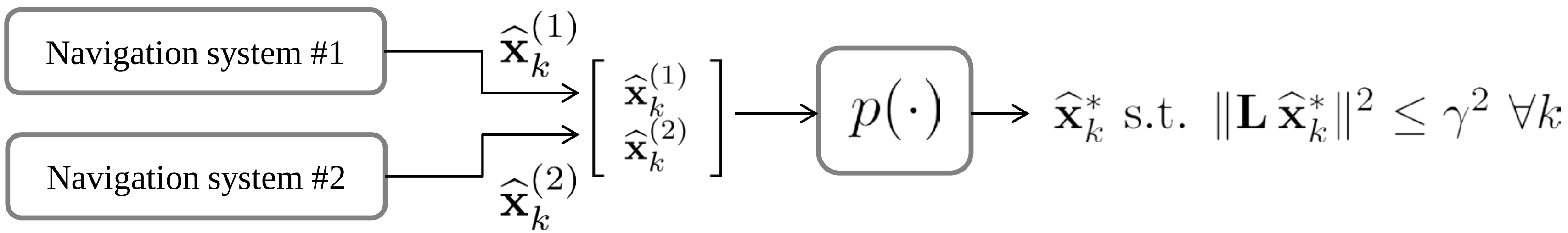
Let

$\mathbf{x}_k^{(i)} \in \mathcal{R}^{n_i}$ be the true state of the i navigation system, with the first s elements representing the position,

$\mathbf{x}_k = [(\mathbf{x}_k^{(1)})^T (\mathbf{x}_k^{(2)})^T]^T \in \mathcal{R}^{n_1+n_2}$ the true joint navigation state.

Then, if their is a upper bound γ on the spatial separation,
$\|\mathbf{L}\mathbf{x}_k\|^2 \leq \gamma^2 \ \forall k$, where $\mathbf{L} = [\mathbf{I}_s \quad \mathbf{0}_{s,n_1-s} \quad -\mathbf{I}_s \quad \mathbf{0}_{s,n_2-s}]$

Hence, we would like the (joint) estimate $\widehat{\mathbf{x}}_k$ *to also fulfill this constraint.*



Proposed solution

If $\|\mathbf{L}\,\widehat{\mathbf{x}}_k\|^2 > \gamma^2$ project onto the subspace $\{\mathbf{x} \in \mathcal{R}^{n_1+n_2} : \|\mathbf{L}\mathbf{x}\|^2 \leq \gamma^2\}$.

One such projection is

$$p(\widehat{\mathbf{x}}_k) \stackrel{\text{def}}{=} \underset{\mathbf{x}}{\operatorname{argmin}}\left(\|\widehat{\mathbf{x}}_k - \mathbf{x}\|^2_{\mathbf{P}_k^{-1}}\right) \quad \text{s.t.} \quad \|\mathbf{L}\,\mathbf{x}\|^2 \leq \gamma^2,$$

where

$$\|\widehat{\mathbf{x}}_k - \mathbf{x}\|^2_{\mathbf{P}_k^{-1}} = (\widehat{\mathbf{x}}_k - \mathbf{x})^T\,\mathbf{P}_k^{-1}(\widehat{\mathbf{x}}_k - \mathbf{x}),$$

and $\mathbf{P}_k$ is the covariance matrix of the joint navigation solution $\widehat{\mathbf{x}}_k$.

The projection is the solution to an inequallity constraint weighted least squares problem.

Solving the constraint LS problem (1)

The solution is a stationary point of the Lagrange function

$$J(\mathbf{x}, \lambda) \stackrel{\text{def}}{=} \|\widehat{\mathbf{x}}_k - \mathbf{x}\|^2_{\mathbf{P}_k^{-1}} + \lambda\, \psi(\mathbf{x}),$$

where

$$\psi(\mathbf{x}) \stackrel{\text{def}}{=} \|\mathbf{L}\,\mathbf{x}\|^2 - \gamma^2.$$

The stationary points are given by the solutions to the normal equations

$$\begin{aligned}
\frac{\partial J(\mathbf{x}, \lambda)}{\partial \mathbf{x}} = \mathbf{0} \quad &\leftrightarrow \quad \left(\mathbf{P}_k^{-1} + \lambda \mathbf{L}^T\,\mathbf{L}\right)\mathbf{x} = \mathbf{P}_k^{-1}\,\widehat{\mathbf{x}}_k \\
\frac{\partial J(\mathbf{x}, \lambda)}{\partial \lambda} = 0 \quad &\leftrightarrow \quad \psi(\mathbf{x}) = 0.
\end{aligned}$$

Solving the constraint LS problem (2)

If there is a exist an unique solution to the LS problem, then the stationary point $\{\lambda^*, \mathbf{x}^*\}$ of the Lagrange function that corresponds to the solution to the constrained least squares problem, is the unique stationary point for which $\lambda > 0$. That is

$$\lambda^* = \{\lambda \in \mathcal{R}^+ : \| \left(\mathbf{P}_k^{-1} + \lambda \, \mathbf{L}^T \mathbf{L}\right)^{-1} \mathbf{P}_k^{-1} \, \widehat{\mathbf{x}}_k \|^2 - \lambda^2 = 0\}.$$

This is a nonlinear polynomial function in λ, and to find its roots, one must in most cases resort to some numerical method such as Newtons method.

Given λ^*, then the projected state estimate $\widehat{\mathbf{x}}_k^* = \left(\mathbf{P}_k^{-1} + \lambda^* \, \mathbf{L}^T \mathbf{L}\right)^{-1} \mathbf{P}_k^{-1} \, \widehat{\mathbf{x}}_k$

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Approximating the covariance of the constraint estimate

The covariance $\mathbf{P}_k^*$ of the projected joint navigation solution $\widehat{\mathbf{x}}_k^*$ can be approximated as

$$\mathbf{P}_k^* = \nabla p \, \mathbf{P}_k (\nabla p)^T,$$

where ∇p is the Jacobian matrix of the projection function $p(\mathbf{x})$.

That is

$$\nabla p = \left(\mathbf{I}_m - \frac{(\mathbf{P}_k^{-1} + \lambda^* \mathbf{L}^T \mathbf{L})^{-1} \mathbf{z}_k \mathbf{z}_k^T}{\mathbf{z}_k^T (\mathbf{P}_k^{-1} + \lambda^* \, \mathbf{L}^T \mathbf{L})^{-1} \mathbf{z}_k} \right) (\mathbf{P}_k^{-1} + \lambda^* \mathbf{L}^T \mathbf{L})^{-1} \mathbf{P}_k^{-1}$$

where $\mathbf{z}_k = \mathbf{L}^T \mathbf{L} \, p(\widehat{\mathbf{x}}_k)$

Summary of the propossed method

1. Construct the joint state vector $\widehat{\mathbf{x}}_k$ and joint covariance matrix $\mathbf{P}_k$ from the navigation solutions of the two subnavigation systems.

2. Enforce the constraint by projecting the joint navigation solution onto the feasaible subspace. That is

   (a) Set up the Lagrange cost function for the IWLS problem.

   (b) Solve the corresponding normal equations for the unique positive Lagrange multiplier λ^*.

   (c) Calculate the solution $\widehat{\mathbf{x}}_k^*$ to the IWLS problem.

3. Calculate the covaraince $\mathbf{P}_k^*$ of the constraint joint navigation solution.

Note: $\widehat{\mathbf{x}}_k^*$ and $\mathbf{P}_k^*$ are a quite crude approximation of the first and second order moment of the projected navigation solution and should be used with care.

Handling navigation systems with attitude estimates

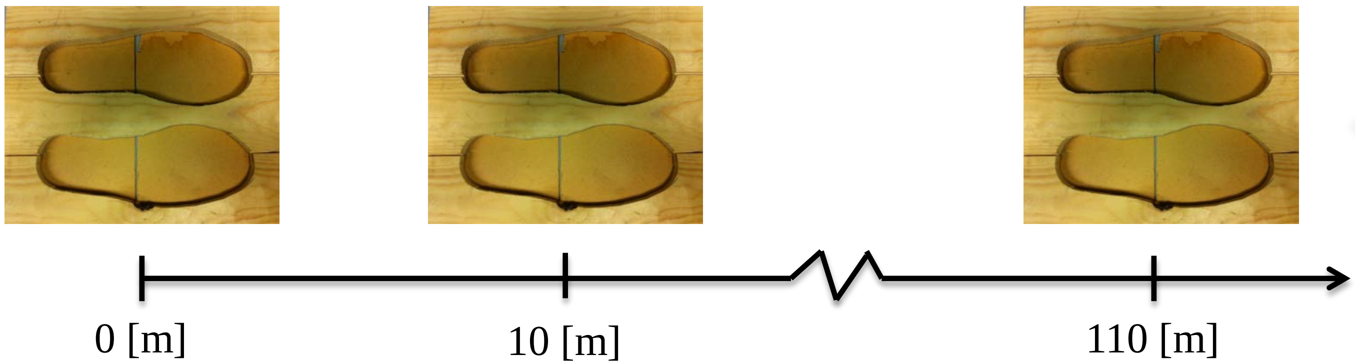
Problem:
The method proposed assumed that $\mathbf{x}_i \in \mathcal{R}^{n_i}$. The attitude states are defined on $[0, 2\pi)$ and a sequency of rotations does not commute $\rightarrow$ the attitude cannot be represented by a "proper" vector

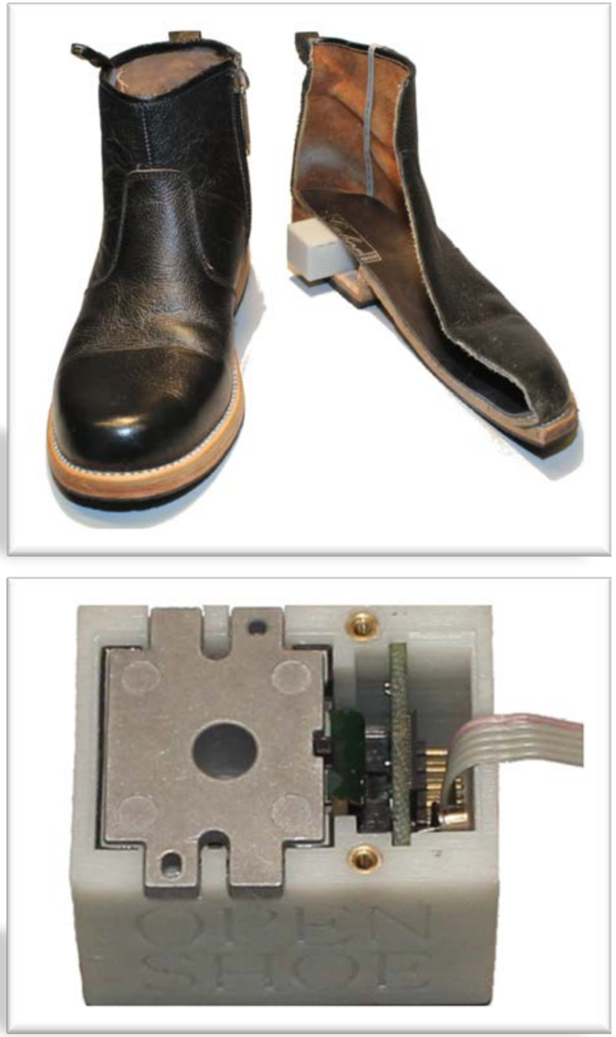
Solution:

$$\begin{bmatrix} \widehat{\mathbf{p}}_k^{(1)} \\ \widehat{\mathbf{v}}_k^{(1)} \\ \widehat{\psi}_k^{(1)} \\ \widehat{\mathbf{p}}_k^{(2)} \\ \widehat{\mathbf{v}}_k^{(2)} \\ \widehat{\psi}_k^{(2)} \end{bmatrix} \xrightarrow{(1)} \begin{bmatrix} \widehat{\mathbf{p}}_k^{(1)} \\ \widehat{\mathbf{v}}_k^{(1)} \\ \mathbf{0} \\ \widehat{\mathbf{p}}_k^{(2)} \\ \widehat{\mathbf{v}}_k^{(2)} \\ \mathbf{0} \end{bmatrix} \xrightarrow{(2)} \text{Proj.} \xrightarrow{(3)} \begin{bmatrix} \widehat{\mathbf{p}}_k^{(1),*} \\ \widehat{\mathbf{v}}_k^{(1),*} \\ \epsilon_k^{(1)} \\ \widehat{\mathbf{p}}_k^{(2),*} \\ \widehat{\mathbf{v}}_k^{(2),*} \\ \epsilon_k^{(2)} \end{bmatrix} \xrightarrow{(4)} \widehat{\psi}_k^{i,*} = \Upsilon(\widehat{\psi}_k^i, \epsilon_k^i) \xrightarrow{(5)} \begin{bmatrix} \widehat{\mathbf{p}}_k^{(1),*} \\ \widehat{\mathbf{v}}_k^{(1),*} \\ \widehat{\psi}_k^{(1),*} \\ \widehat{\mathbf{p}}_k^{(2),*} \\ \widehat{\mathbf{v}}_k^{(2),*} \\ \widehat{\psi}_k^{(2),*} \end{bmatrix}$$

Experiment

- A user was equipped two OpenShoe navigation system and asked to walk along a strait line for 110 m
- As reference points plates with imprints of the shoes were positioned at 0[m], 10[m], and 110[m].
- Twenty trajectories with 4 different OpenShoe units were collected.
- The data was the processed with the proposed method.





The OpenShoe navigation system

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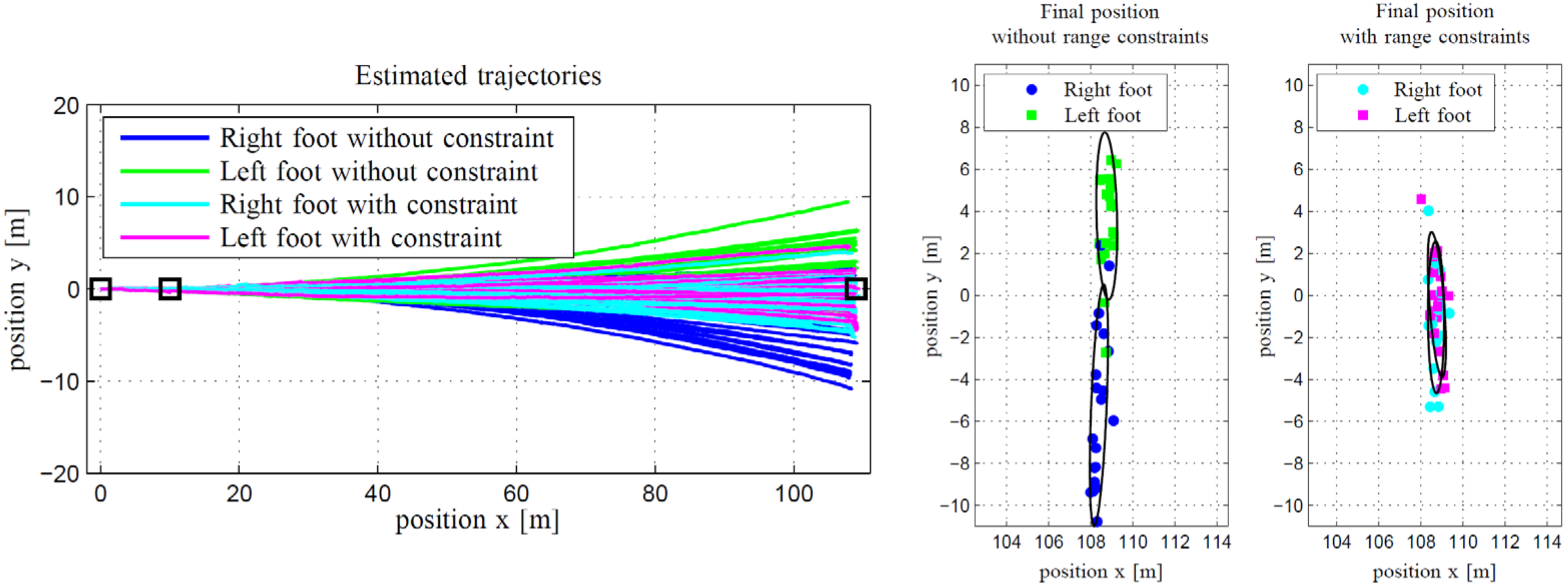
Results



Reproducible Research: The data and Matlab code used in this paper are available at www.openshoe.org.

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Conclusions

- A method to fuse the navigation solution from two navigation system, when there is an upper limit on their maximum spatial seperation has been proposed.
- The proposed method has been applied to two foot-mounted zero-velocity aided INS, and tested using real world data.
- The results indicates that the method can reduce final position error significantly

Bonus:

- You may try the OpenShoe system with the propossed method at demo session.
- A more statistically correct method can be found in:

Zachariah, D.; Skog, I.; Jansson, M.; Händel, P.; , "Bayesian Estimation With Distance Bounds," Signal Processing Letters, IEEE , vol.19, no.12, pp.880-883, Dec. 2012

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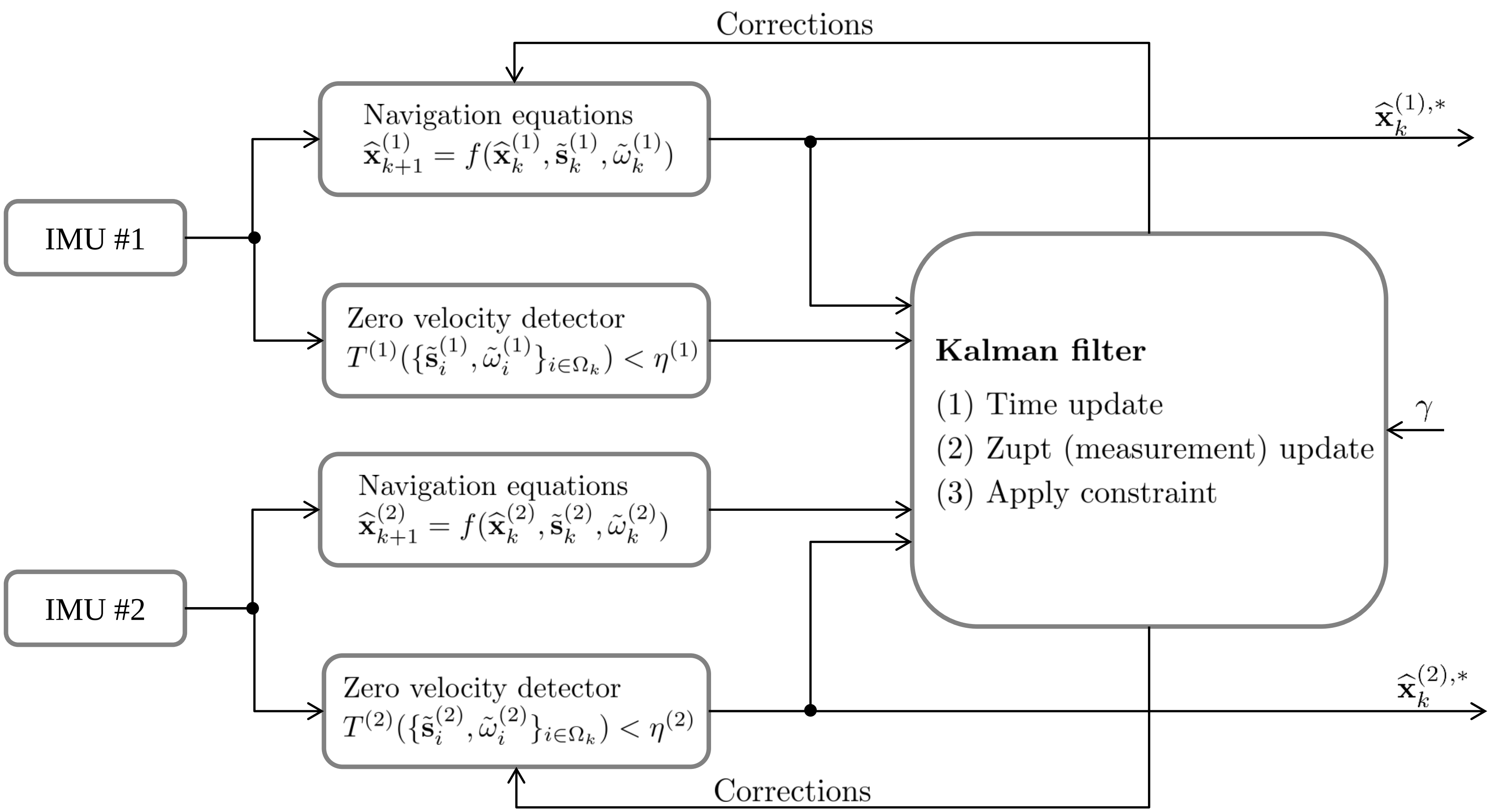
Appling the method to two foot-mounted zero-velocity aided INSs



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Pseudo code

Algorithm 1 Pseudo code for the proposed Kalman filter algorithm.

$k \leftarrow 0,\ c_Z \leftarrow -\tau_Z$
$\widehat{\mathbf{x}}_k \leftarrow$ **Process**{ Joint initial navigation state }
$\mathbf{P}_k \leftarrow$ **Process**{ Initial covariance matrix }
loop
  $k \leftarrow k+1$
  $[\widehat{\mathbf{x}}_k]_{1:9} \leftarrow f([\widehat{\mathbf{x}}_{k-1}]_{1:9}, \widetilde{\mathbf{s}}_k^{(1)}, \widetilde{\omega}_k^{(1)})$
  $[\widehat{\mathbf{x}}_k]_{10:18} \leftarrow f([\widehat{\mathbf{x}}_{k-1}]_{10:18}, \widetilde{\mathbf{s}}_k^{(2)}, \widetilde{\omega}_k^{(2)})$
  $\mathbf{P}_k \leftarrow \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T + \mathbf{G}_k \mathbf{Q} \mathbf{G}_k^T$
  $T_k^{(1)} \leftarrow$ **Process**{ Zero-velocity detector for system #1 }
  $T_k^{(2)} \leftarrow$ **Process**{ Zero-velocity detector for system #2 }
  if $T_k^{(1)} \leq \eta^{(1)}$ **or** $T_k^{(2)} \leq \eta^{(2)}$ **then**
    $\mathbf{K}_k \leftarrow \mathbf{P}_k \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$
    $\delta\widehat{\mathbf{x}}_k \leftarrow -\mathbf{K}_k \mathbf{H}_k \widehat{\mathbf{x}}_k$
    $[\widehat{\mathbf{x}}_k]_{1:9} \leftarrow \Gamma([\widehat{\mathbf{x}}_k]_{1:9}, [\delta\widehat{\mathbf{x}}_k]_{1:9})$
    $[\widehat{\mathbf{x}}_k]_{10:18} \leftarrow \Gamma([\widehat{\mathbf{x}}_k]_{10:18}, [\delta\widehat{\mathbf{x}}_k]_{10:18})$
    $\mathbf{P}_k \leftarrow (\mathbf{I}_{18} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}_k$
    if $\|\mathbf{L}\widehat{\mathbf{x}}_k\|^2 > \gamma^2$ **and** $k - c_Z > \tau_Z$ **then**
      $\widehat{\mathbf{x}}_k \leftarrow p(\widehat{\mathbf{x}}_k)$
      $\mathbf{P}_k \leftarrow \nabla p\, \mathbf{P}_k (\nabla p)^T$
      $c_Z \leftarrow k$
    end if
  end if
end loop

Notation

$(\cdot)^{(i)}$	Superscript indicating a quantity related to subsystem i
k	Time index
τ_z	Value controlling the rate at which the constraint is applied
$[\mathbf{a}]_{i:j}$	Element i to j of vector $\mathbf{a}$
$\delta\mathbf{a}$	Perturbation of vector $\mathbf{a}$
$f(\cdot)$	Navigation equations
$\Gamma(\cdot)$	Function that given the state perturbations corrects the state vector
$p(\cdot)$	Projection operator
∇p	Gradient of the projection operator
T	Detector test statistics
η	Detector threshold
γ	Range constraint
$\tilde{\mathbf{s}}_k$	Specific force measurement
$\tilde{\omega}_k$	Angular rate measurement
$\mathbf{I}$	Identity matrix
$\mathbf{F}_k$	Joint state transition matrix
$\mathbf{H}_k$	Joint measurement matrix
$\mathbf{G}_k$	Joint process noise gain matrix
$\mathbf{Q}_k$	Joint process noise covariance matrix
$\mathbf{R}_k$	Joint measurement noise covariance matrix
$\mathbf{P}_k$	Joint state covariance matrix