

# A note on the limitations of ZUPTs and the implications on sensor error modeling

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**Abstract**—The limitations of zero-velocity-updates (ZUPTs) for aiding a foot-mounted inertial navigation system (INS) are studied. Multiple significant modeling errors related to the ZUPTs are pointed out and quantified. Their implications for the possibility to estimate systematic inertial sensor errors are discussed and it is argued that modeling and estimating such errors, in foot-mounted ZUPT-aided INSs, should be avoided in many cases.

## I. INTRODUCTION

Zero-velocity updates (ZUPTs) combined with a foot-mounted inertial navigation system (INS) have been shown to have the capability to provide accurate and reliable pedestrian dead-reckoning. Unfortunately, due to the integrative nature of the system, errors will accumulate giving an unbounded error growth. In principle, there are two sources of errors which will enter the system, measurement errors and modeling errors. To reduce the growth rate and improve the performance, systematic measurement errors can be modeled and estimated. However, if the modeled errors do not have a dominating effect on the observations in the system, this might just add more modeling errors and lead to worse performance.

In ZUPT-aided INSs inertial sensor bias errors are typically modeled, e.g. [1][2][3][4][5][6]. However, it is far from clear that the bias errors have a dominating effect on the errors of the ZUPTs. We believe that this is often not the case and that modeling errors in the system, especially of the ZUPTs themselves, have an equal or larger effect. The implications of this is that modeling systematic sensor errors should often be avoided. The ZUPTs are simply too poor in quality for estimating systematic sensor errors of many modern MEMS inertial measurement units (IMUs) used in these systems. This advice is in strong contrast to the large number of publications presenting systems in which sensor bias errors are modeled. It should be noted that, as far as we know, there are no publications showing that modeling sensor biases improves the performance of presented systems and our own experience is that in most cases it does not. Therefore, in this short article we will point out and quantify various modeling errors and discuss their implications for modeling and estimating sensor errors in foot-mounted ZUPT-aided INSs.

All of the data presented in this article have been collected with OpenShoe units that were mounted below the heels. See [www.openshoe.org](http://www.openshoe.org) or [7] for further details. The units were oriented such that the  $x$ -axis points in medial direction, the  $y$ -axis points forward, and the  $z$ -axis points downward. The

processing has been done as described in [7]. Most of the presented errors will change with the IMU, the mounting point, the filter settings, the positioned agent, and the external environment. Consequently, the errors measured by a different system and under different conditions might be different. However, many characteristics will most likely be the same.

## II. MODELING ERRORS

ZUPTs are occurrences of stationarity detected by some hypothesis testing [8]. The ZUPTs pose somewhat of a clash between detection theory and Bayesian filtering. The detection is based on a statistical measure of how likely a hypothesis is in relation to a null-hypothesis. On the other hand, the Kalman type of filter, which is typically used for ZUPT-aided INSs, implicitly makes the assumption that the velocity errors (filter innovations) provided by the ZUPTs are zero-mean and white as well as independent of the state estimates. The detection says nothing about this and, in reality, these assumptions will seldom hold perfectly. Consequently, in the following subsections we will point out and quantify various related modeling errors showing that there are indeed significant modeling errors in the system.

### A. Rotations during the stance phase

Some publications studying the validity of the zero-velocity assumptions using camera tracking systems exists [9][4]. These studies both suggest that there are systematic motions during the stance phases while walking. Here we point out another indicator of the same phenomena. Even though we do not make zero-angular rate updates, the angular rates should be small during a stationary period. Fig. 1 shows the mean angular rates over some 60 steps, relative to the first ZUPT (time 0[s]) of a stance phase. From the zoomed out perspective of the upper plot, the angular rates over the period between 0 and 0.24[s] (the period which is normally declared stationary) seem small. However, the zoomed in view of the lower plot reveals that the angular rates during this period are clearly non-zero. Most likely, the center of rotation is not the IMU mounted in the heel. This implies that there is motion during the stationary periods. The smallest angular rate is around a few  $^{\circ}/s$ . With a few [dm] lever arm, this would give a speed of a few [mm/s]. This is in agreement with the findings in [9][4] and indicates that the non-zero velocity may primarily be caused by a rotation of the foot during the stance phase. The

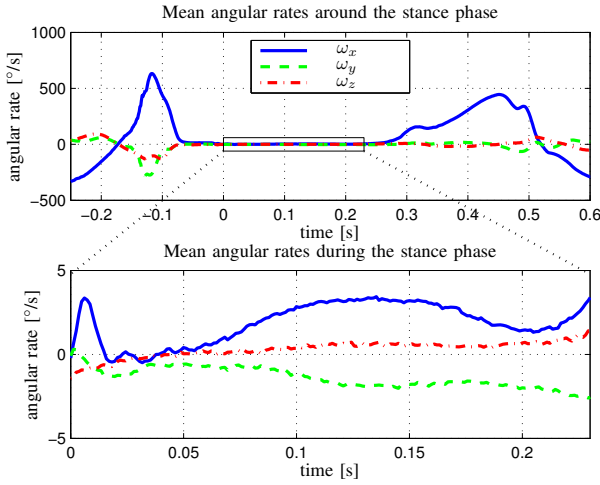


Fig. 1: The mean angular rates following the first ZUPT (time 0[s]) in the stance phase of a step during walking. The period between 0 and 0.24[s] is the period which is normally detected as stationary. Clearly there are systematic non-zero rotations during this period. Note that the height of the zoom-box in the upper plot is exaggerated.

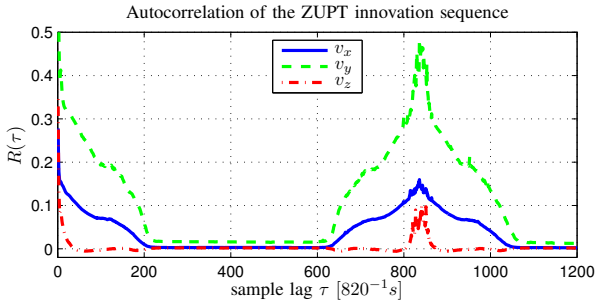


Fig. 2: The figure shows the autocorrelation of the ZUPT innovation sequence from a 400[m] straight-line walk. The innovations are seen to be correlated within steps (lag 0-220) and between steps (lag 630-1070). This is in contrast to the fundamental assumptions of the Kalman filter.

non-zero angular rates during the declared stationary periods clearly give systematic modeling errors of the ZUPTs.

### B. Correlated innovation sequences

An indicator of the quality of the zero-velocity assumption, as seen by the filtering, is the innovation sequences. These sequences should be zero-mean and white. By construction of the ZUPTs, we do not expect the sequence to be white. However, the correlations should die out reasonable fast. Fig. 2 shows the autocorrelations of the  $x$ ,  $y$ , and  $z$  components of the innovation sequence, resulting from the ZUPTs during a 400[m] straight-line walk at a normal gait speed. Clearly, the innovations over a stationary period are correlated. What is worse is that the innovations from adjacent steps are also correlated. This could be interpreted to be due to measurement biases. However, prior to recording the data, care has been

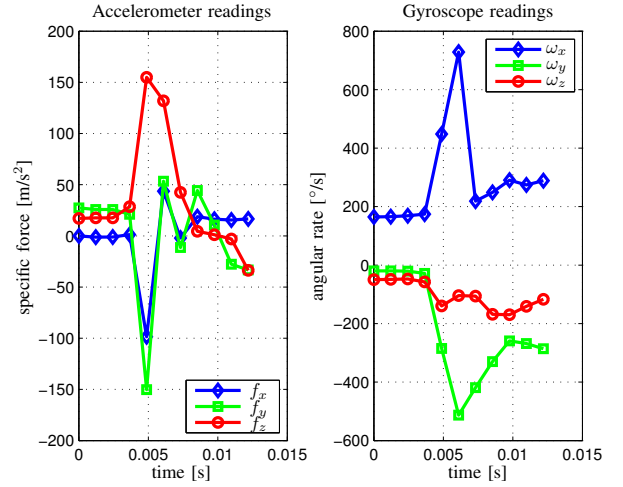


Fig. 3: IMU measurements showing the impact of the foot on the ground during normal walking. Clearly the bandwidth of the sensors are insufficient to capture the dynamic of the impact.

taken to calibrate and compensate for sensor biases. Therefore, we rather see them as indicators of modeling errors (apart from sensor biases) in the system.

### C. Insufficient bandwidth

One potential reason for the correlated innovation sequences are other systematic inertial measurement errors apart from the biases. This could give a systematic error prior to the ZUPTs of each stance phase. A potential source of such errors is the finite bandwidth of the sensors. The foot-mounted IMU will be exposed to quite extreme dynamics when the heel hits the ground just prior to the stance phase. Here the bandwidth and the dynamic range of most IMUs are not sufficient. The impact as seen from an OpenShoe unit mounted in the sole is shown in Fig. 3. The sampling rate of 820[Hz] and the bandwidth of 330[Hz] are clearly seen to be insufficient despite them being on the upper end of IMU used for the current application. The sensors readings stay within the dynamic range of 18[g] and 1200[°/s], but most likely this is due to the insufficient bandwidth. Since the impact will have similar characteristics for each step, the insufficient bandwidth will add systematic measurement/modeling errors in the system. This is probably a contributing factor to the correlated innovation sequences.

### D. Systematic heading drift

As we have seen by studying the errors of individual ZUPTs and periods of ZUPTs, it is clear that there are modeling errors. Unfortunately, directly relating these error to large-scale dead-reckoning performance is difficult. However, some large-scale errors which arise in the system can be easily observed. In Fig. 4, 20 straight-line trajectories, as estimated from two OpenShoe units placed on the right foot, are shown in green and 20 trajectories, from two Openshoe unit placed on the left foot, are shown in red. As a heading reference in the beginning of the trajectory, two plates with imprints of the

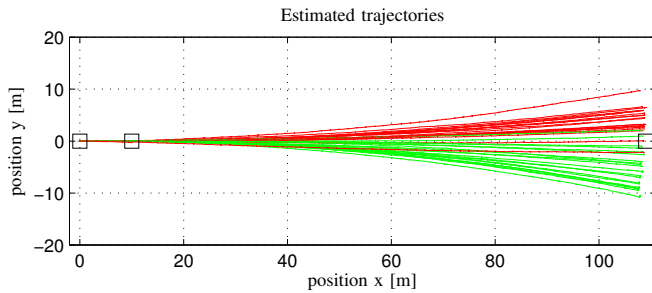


Fig. 4: The estimated trajectories from walking along a 110[m] straight line. The green lines indicate the estimated trajectories from an IMU mounted on the right foot and the red lines from an IMU mounted on the left foot. The black boxes indicate the location of the starting position (0[m]), the heading reference point (10[m]), and the stop position (110[m]).

shoes distanced 10[m] apart were used. These are indicated with the black squares in the figure. The individual trajectories were rotated so that the position at the second plate was located on the horizontal axis. Prior to each trajectory, the systems were held stationary for 20[s] and the mean gyro reading was subtracted from the subsequent readings to remove any gyro bias errors. The gyro biases were also verified with a stationary period at the end of each trajectory and no significant errors were found. Despite this, systematic heading drifts are clearly seen. Naturally, the drifts are symmetrical for the right and the left foot. These errors are large-scale manifestations of modeling errors in the system.

#### E. Step length underestimation and step height overestimation

Another noticeable large-scale error similar to the systematic heading drift is a systematic underestimation of the stride length and overestimation of the step height. Based on the measurements in [9], it is suggested that the stride length should be underestimated. This has indeed been observed. Similar behavior can also be noted in the data published in [10] and [11].

The zero-velocity is detected if the test statistic is below a threshold. This threshold will significantly effect the performance as shown in [12]. The threshold which gives the best performance is significantly above the test statistic of a stationary system. This is due to the fact that the system is never perfectly stationary during the stance phase of the gait. However, we still have to make some updates to restrain the system. This has the consequence that the steps are, in a sense, “cut short” at both ends. The beginning and the end of a step are declared stationary and the related motion ignored. The situation gets worse if the system needs to operate for a running user. In this case the threshold needs to be increased to get any updates resulting in more errors for a walking user. The mean final position errors as a function of the threshold and for different IMUs and different feet are shown in Fig. 5. The different IMUs where mounted in different shoes which might explain some of the systematic differences. However, the variations with the detection threshold indicate that the errors

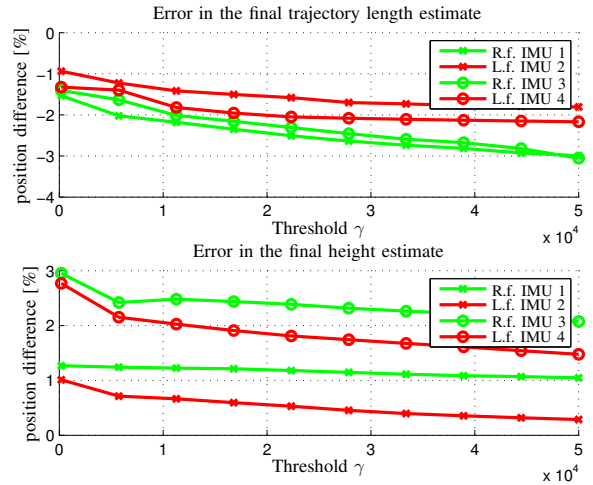


Fig. 5: The relative change of the final position estimate of the trajectories shown in Fig. 4 as a function of the zero-velocity detection threshold  $\gamma$ .

in the trajectory length estimate are caused by the ZUPTs. The incorrect height estimate gets better with a high threshold. This may indicate that this error is caused by the limited bandwidth and the impact of the foot on the ground. The acceleration there could be underestimated and the sooner the motion is clipped (higher threshold), the less the error will be.

### III. SENSOR ERROR MODELING

The sensor error modeling and estimation for ZUPT-aided INSs is a heritage from the predating active field of low-cost GPS-aided INS. Indeed, the basic error model and the filter setup of the seminal paper [1] is pretty much just a copy-and-paste from the predominant setup of GPS-aided INS of that era [13]. Taking the setup from the GPS-aided INS seems reasonable considering that similar inertial sensors were used. However, the capability of ZUPT-aiding to estimate sensor bias errors is quite different from that of a GPS-aiding. With GPS-aiding, the INS is aided by position observations. Between the position estimates and the angular rate and specific force measurements, there are three and two integrations respectively. In contrast, with ZUPT-aiding, the INS is aided by velocity pseudo-measurements. Between the velocity estimates and the aforementioned inertial measurements, there are two and one integrations respectively. Consequently, the effect of the systematic inertial measurement errors on the velocity estimates is smaller when compared with the position estimates. Therefore, the conditions for estimating systematic measurement errors are worse in the ZUPT-aided case when compared with the GPS-aided case.

Of course, the conditions for estimating the sensor errors are relative to the quality of the observations used to estimate them. The quality of the ZUPTs is mainly a consequence of the gait and the environment. The short window size found to give the best performance in [12] indicates that the IMU

performance is of less significance when it comes to producing ZUPTs. The quality of the ZUPTs will not change with time. The sensor errors though, we can expect to change with time (get better with time) and with the price-tag (get better with a higher price). The feasibility of estimating sensor errors are dependent on the magnitude relation between the effects of them and the ZUPT modeling errors. Consequently, even though once upon a time it might have been necessary and beneficial to estimate sensor errors for foot-mounted ZUPT-aided INSs, with many modern sensors this might not be the case. The quality of the sensors has simply improved while the quality of the ZUPTs remains the same. It should be noted that ZUPTs triggered by some external measurements as suggested in [4][14] will not necessarily improve the situation, even though it will most likely improve the robustness. The foot is simply not stationary. Indeed in [12] it is found that the pressure sensors do not give an improved performance.

In conclusion, the situation for estimating systematic inertial sensor errors based on ZUPTs is poor. It is really a matter of how large the systematic sensor errors are. If they are large when compared with the modeling errors in the system, one could potentially improve the performance by modeling them. This might be the case for emerging single-chip IMUs but so far we are not aware of any foot-mounted ZUPT-aided INS built around such sensors. For the high end and medium range research MEMS IMUs typically seen in the presented systems, we believe that sensor errors should not be estimated. One should definitely resist the temptation of using more complicated modeling for the sake of it. Also in determining if bias errors should be modeled, one should be careful. As it has been shown, the modeling errors give rise to systematic dead-reckoning errors which could be mistaken for sensor bias errors. However, these errors will change with gait style and the environment. Therefore, in a too clean test setup, in a uniform environment with a uniform gait, modeling bias errors could improve performance, just to give a worse performance in a more realistic scenario with a varying gait and environment. Ideally a scenario based evaluation should be used [15].

#### IV. WHAT TO DO

There will be systematic sensor errors as well as modeling errors. The question is, what should be done about them if we cannot model and estimate them with the ZUPTs. Fortunately there are some things that can be done to minimize them. The gyro errors can be estimated during long stationary periods and compensated in an outer loop. Foot-mounted INS can be used on both feet as suggested in [16][17][18]. This way symmetrical modeling errors will cancel out. Possibly the mounting point of the IMU could be altered to minimize the modeling errors. Below the forefoot seems like a good mounting point, but it remains to be tested. At some point we also have to realize that ZUPT-aided INS has some fundamental limitations and is not going to get significantly better beyond a point unless additional measurements are added. This is often seen

in the literature, but it does not change the conditions for modeling and estimating systematic sensor errors while using ZUPTs.

#### V. CONCLUSIONS

Be careful with modeling and estimating sensor errors for foot-mounted ZUPT-aided INSs. What you are estimating might not be what you think.

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