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# Foot-mounted zero-velocity aided inertial navigation

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# Course Outline

## **1. Foot-mounted inertial navigation**

- a. Basic idea
- b. Pros and cons

## **2. Inertial navigation**

- a. The inertial sensors
- b. The navigation equations
- c. Error propagation

## **3. Kalman filtering**

- a. Direct filtering
- b. Complimentary filtering
- c. Pseudo observations and motion constraints

## **4. Zero-velocity detection**

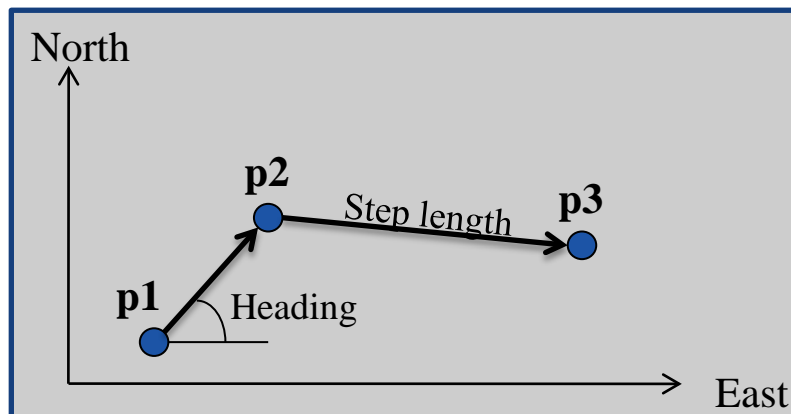
- a. Force sensitive resistors
- b. The SHOE detector
- c. Characteristics

## **5. The OpenShoe project**

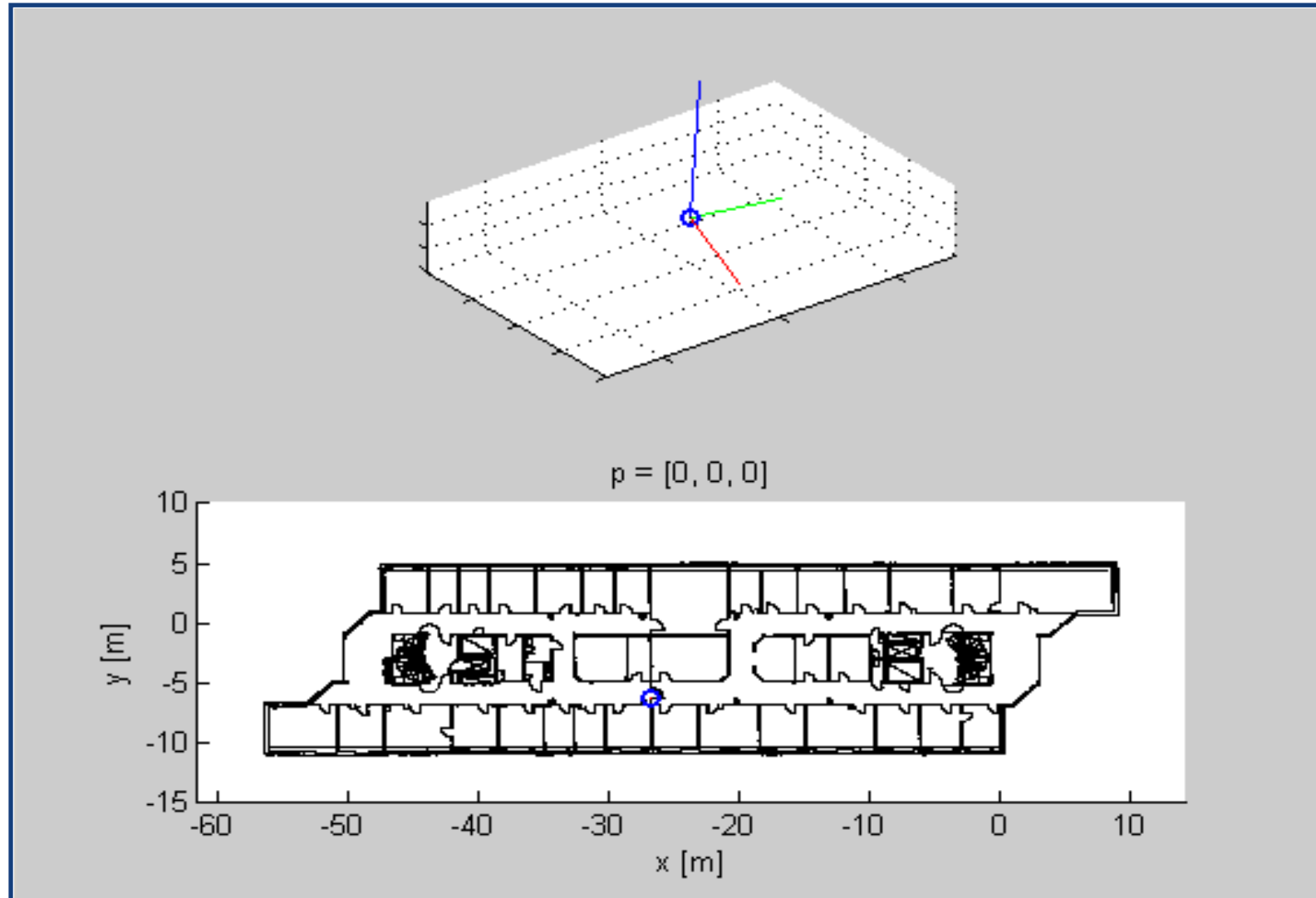
- a. Background and Goal
  - b. Hardware/Software
  - c. Demo
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# Basic idea behind foot-mounted inertial navigation

1. Mount (inertial) sensors in the sole of the shoes of a user
2. Measure the length and direction of the steps the users takes
3. Calculate the change in position via dead-reckoning.



# Example: Output from a foot-mounted inertial navigation system



# Pros and cons of foot-mounted inertial navigation

- **Pros**

- Does not depend on any preinstalled infrastructure.
- Can not be disturbed.
- No motion constraints\*.

- **Cons**

- Initial position and heading must be known.
- The position and heading error grows with time.

\* We assume that the foot becomes stationary on a regular basis

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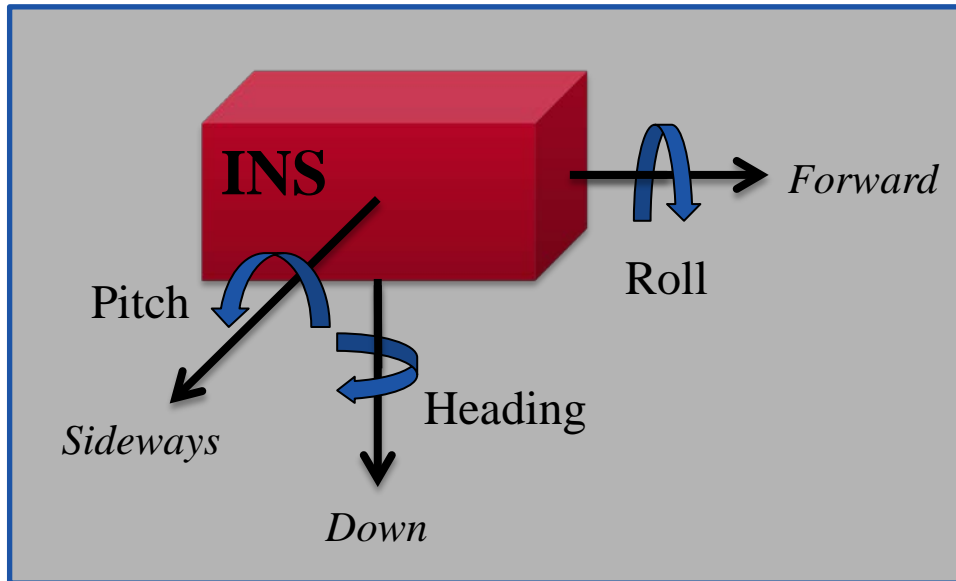
# Inertial navigation

- Coordinate systems and rotations
- Sensors
- Navigation equations
- Error propagation

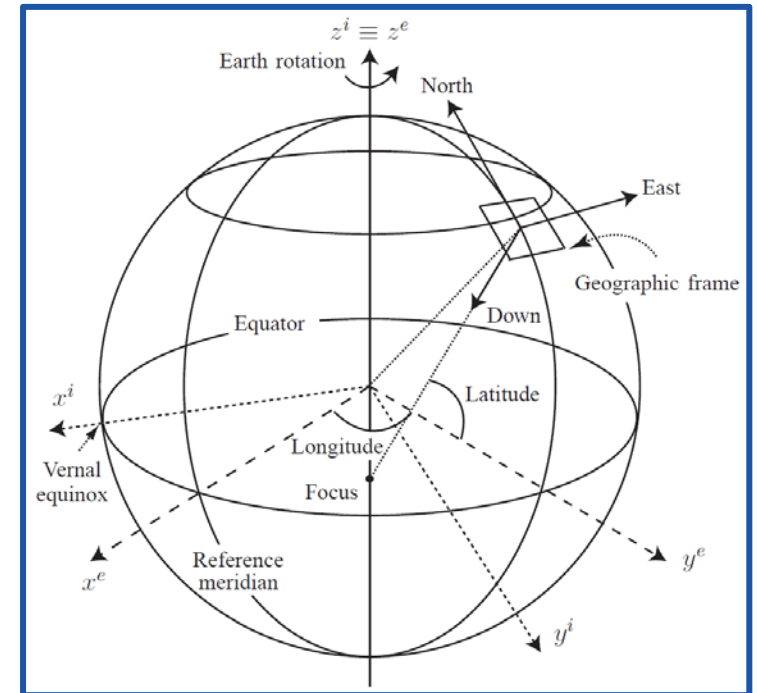
# Notation

Description	Notation	
Discrete time index	$k$	
Sampling period	$T_s$	
Scalar	$a$	
Vector (matrix)	$\mathbf{a}$ ( $\mathbf{A}$ )	
Identity matrix	$\mathbf{I}$	
Gravity vector	$\mathbf{g}$	
Natural basis vector $\ell$	$\mathbf{e}_\ell$	
Transpose	$(\cdot)^T$	
Quantity expressed in coord. system $j$	$(\cdot)^j$	$j = \{e, n, b, i\}$
$\ell$ :th component of a vector	$[\mathbf{a}]_\ell$	
Rotation matrix	$\mathbf{R}_b^n$	
Cross product	$\mathbf{a} \times \mathbf{b}$	
Cross product matrix	$[\mathbf{a}]_\times$	$[\mathbf{a}]_\times \mathbf{b} = \mathbf{a} \times \mathbf{b}$
Perturbation	$\delta \mathbf{a}$	
Estimated quantity	$\hat{\mathbf{a}}$	$\hat{\mathbf{a}} = \mathbf{a} + \delta \mathbf{a} \quad \hat{\mathbf{R}}_b^n = (\mathbf{I} - [\epsilon]_\times) \mathbf{R}_b^n$
Measured quantity	$\tilde{\mathbf{b}}$	$\tilde{\mathbf{b}} = \mathbf{b} + \delta \mathbf{b}$

# Coordinate systems and rotations



Body coordinate system and the three Euler angles.

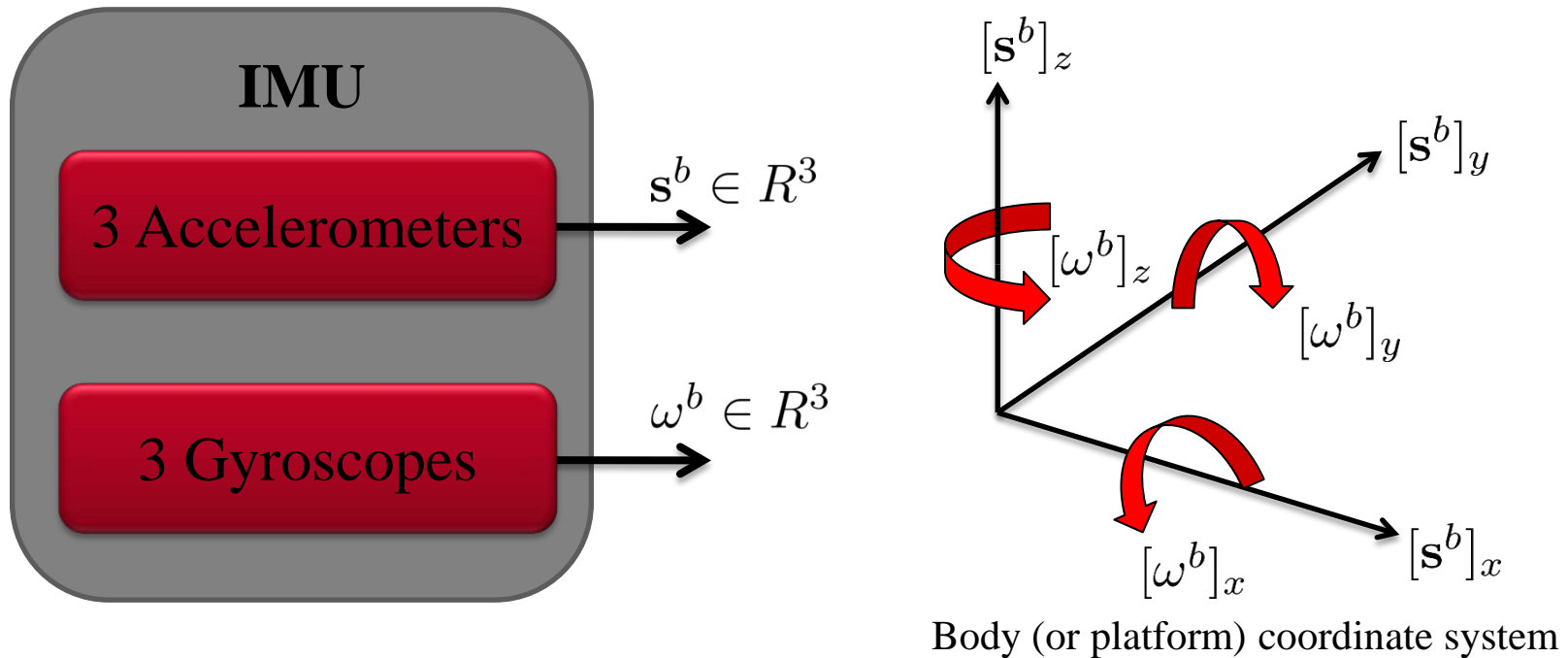


Earth centered inertial, earth centered earth fixed, and geographic coordinate system.

## Orientation representation

- Euler angles (roll, pitch, yaw)  $\theta$
- Rotation matrix  $\mathbf{R}_b^n$ ,  $\mathbf{a}^n = \mathbf{R}_b^n \mathbf{a}^b$
- Quaternion vector  $\mathbf{q}$

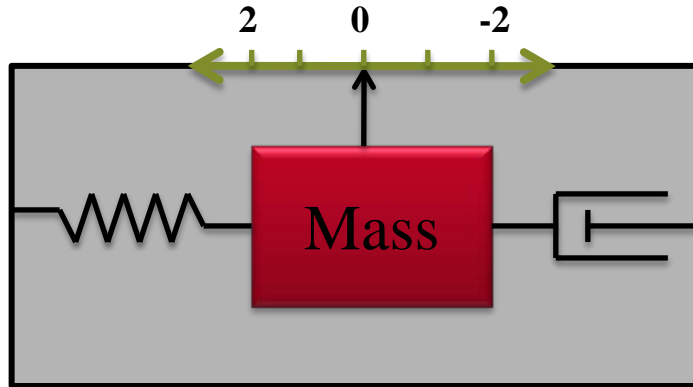
# Inertial measurement unit (IMU)



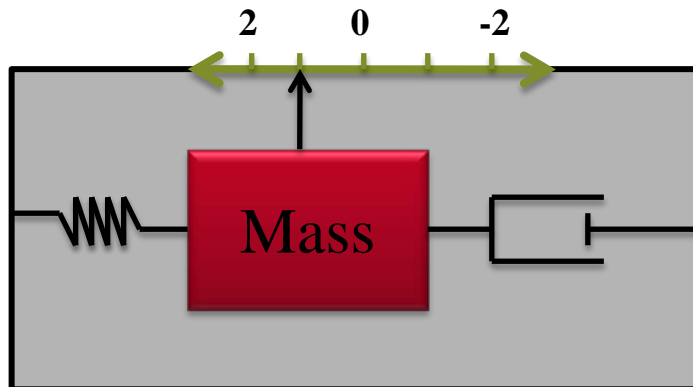
## Minimum requirements

- Sampling rate 100 Hz
- Accelerometer dynamic range  $\pm 15g$
- Gyroscope dynamic range  $\pm 1200^\circ/s$

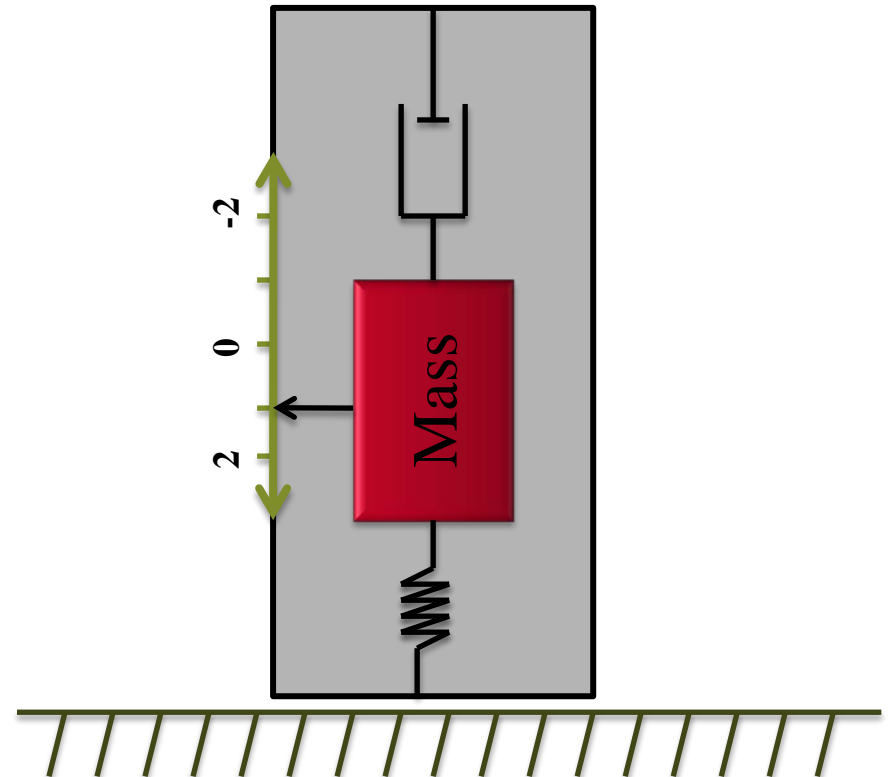
# The accelerometer and its output (1)



Stationary accelerometer.



Accelerometer accelerating to the right, and with the sensitivity axis orthogonal to the gravity field.

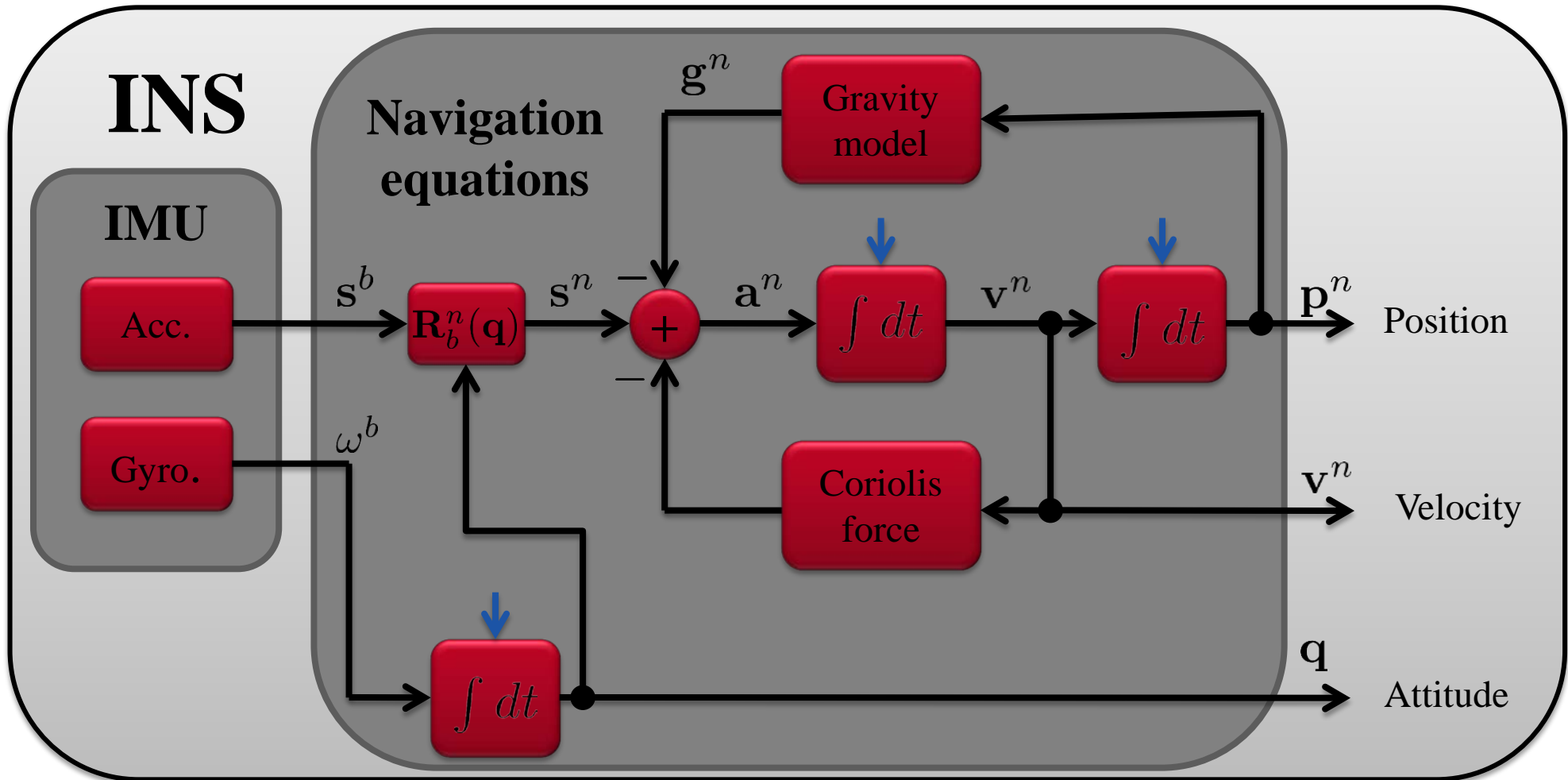


Accelerometer stationary on the earth and with the sensitivity axis aligned with the gravity field.

## The accelerometer and its output (2)

- The inertial and gravitational acceleration are indistinguishable to an accelerometer.
  - The output of an accelerometer is called *specific force*, and includes both the inertial acceleration and the gravity acceleration.
- *To calculate the inertial acceleration from the specific force output of an IMU we must know the IMU's orientation w.r.t. the geographic coordinate system.*

# Inertial navigation system (INS)



# The navigation equations

For a foot-mounted INS using low-cost sensors the following navigation equations can be used.

$$\mathbf{x}(k) = f(\mathbf{x}(k-1), \mathbf{s}^b(k), \omega^b(k)) \quad \mathbf{x}(0) = \{\text{Initial Condition}\}$$

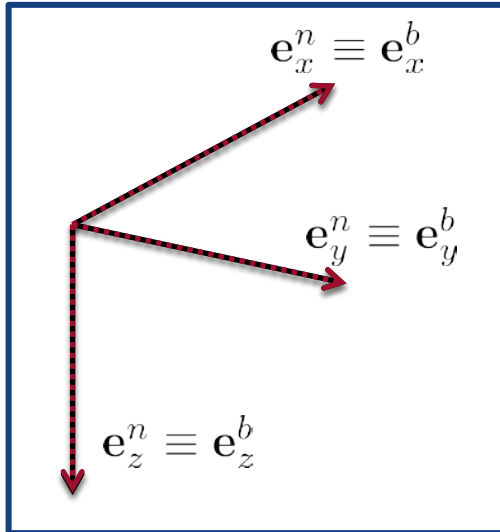
where

$$\mathbf{x}(k) = \begin{bmatrix} (\mathbf{p}^n(k))^T & (\mathbf{v}^n(k))^T & (\mathbf{q}(k))^T \end{bmatrix}^T$$

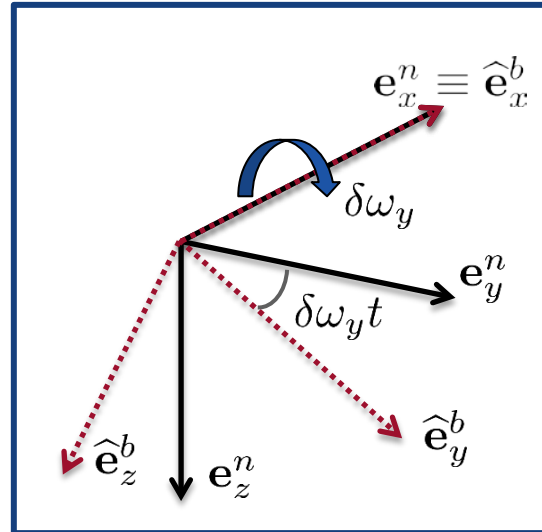
$$f : \begin{cases} \mathbf{p}^n(k) = \mathbf{p}^n(k-1) + T_s \mathbf{v}^n(k) \\ \mathbf{v}^n(k) = \mathbf{v}^n(k-1) + T_s (\mathbf{R}_b^n(\mathbf{q}(k)) \mathbf{s}^b(k) - \mathbf{g}^n) \\ \mathbf{q}(k) = \left( \cos\left(\frac{T_s \|\omega^b(k)\|}{2}\right) \mathbf{I} + \frac{2}{T_s \|\omega^b(k)\|} \sin\left(\frac{T_s \|\omega^b(k)\|}{2}\right) \boldsymbol{\Omega}(k) \right) \mathbf{q}(k-1) \end{cases}$$

$$\boldsymbol{\Omega}(k) = \frac{T_s}{2} \begin{bmatrix} 0 & [\omega^b(k)]_z & -[\omega^b(k)]_y & [\omega^b(k)]_x \\ -[\omega^b(k)]_z & 0 & [\omega^b(k)]_x & [\omega^b(k)]_y \\ [\omega^b(k)]_y & -[\omega^b(k)]_x & 0 & [\omega^b(k)]_z \\ -[\omega^b(k)]_x & -[\omega^b(k)]_y & -[\omega^b(k)]_z & 0 \end{bmatrix}$$

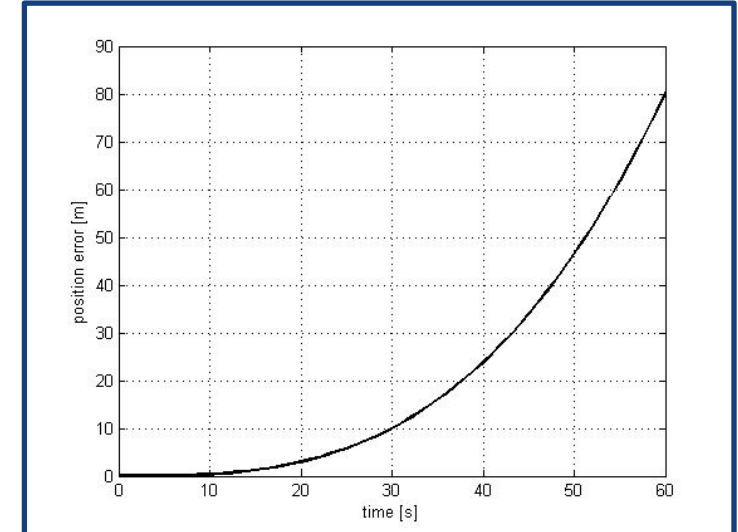
# Error propagation



True orientation of the navigation and body coordinate system.



Orientation of the estimated body coordinate system after  $t$  [s].



Position error as a function of time with a gyroscope bias of 0.013 deg/s.

$$\delta \mathbf{a}^n = \mathbf{a}^n - \hat{\mathbf{a}}^n = \mathbf{0} - (\hat{\mathbf{R}}_b^n \mathbf{s}^b - \mathbf{g}^n) = \mathbf{g}^n - \hat{\mathbf{R}}_b^n \mathbf{R}_n^b \mathbf{g}^n = \mathbf{g}^n - (\mathbf{I} - [\epsilon]_{\times}) \mathbf{R}_b^n \mathbf{R}_n^b \mathbf{g}^n = \begin{bmatrix} 0 \\ -g[\delta\omega]_y t \\ 0 \end{bmatrix}$$

$$\Rightarrow |[\delta \mathbf{p}^n(\tau)]_y| = \left| \int_0^\tau [\delta \mathbf{a}^n]_y dt \right| = \frac{|[\delta\omega^b]_y| g \tau^3}{6}$$

# Error state space model

The error in the foot-mounted INS can be modeled as

$$\delta \mathbf{x}(k) = \mathbf{F}(k) \delta \mathbf{x}(k-1) + \mathbf{G}(k) \mathbf{w}(k)$$

where

$$\delta \mathbf{x}(k) = \begin{bmatrix} (\delta \mathbf{p}^n(k))^T & (\delta \mathbf{v}^n(k))^T & (\epsilon(k))^T \end{bmatrix}^T$$

$$\mathbf{F}(k) = \begin{bmatrix} \mathbf{I} & T_s \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & T_s [\mathbf{s}^n(k)]_{\times} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \quad \mathbf{G}(k) = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{R}_b^n(k) & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_b^n(k) \end{bmatrix}$$

$\mathbf{w}(k)$  - Additive white noise with covariance matrix  $\mathbf{Q}$

Note that  $\mathbf{x}(k) = \Gamma(\hat{\mathbf{x}}(k), \delta \mathbf{x}(k))$ , where  $\Gamma : \begin{cases} \mathbf{p}^n(k) = \hat{\mathbf{p}}^n(k) + \delta \mathbf{p}^n(k) \\ \mathbf{v}^n(k) = \hat{\mathbf{v}}^n(k) + \delta \mathbf{v}^n(k) \\ \mathbf{q}(k) = h_{\mathbf{R}}^{\mathbf{q}}((\mathbf{I} + [\epsilon(k)]_{\times}) \mathbf{R}_b^n(\hat{\mathbf{q}}(k))) \end{cases}$

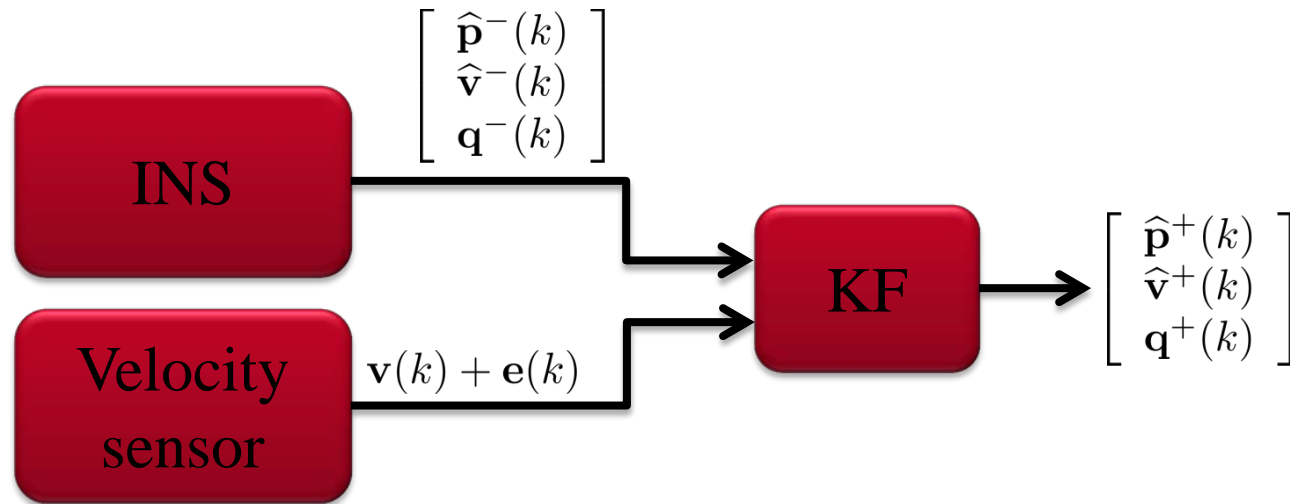


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# Kalman filtering and INSs

- Direct filtering
- Complementary filtering
- Feedback structure
- Pseudo observations and motion constraints

# Direct filtering



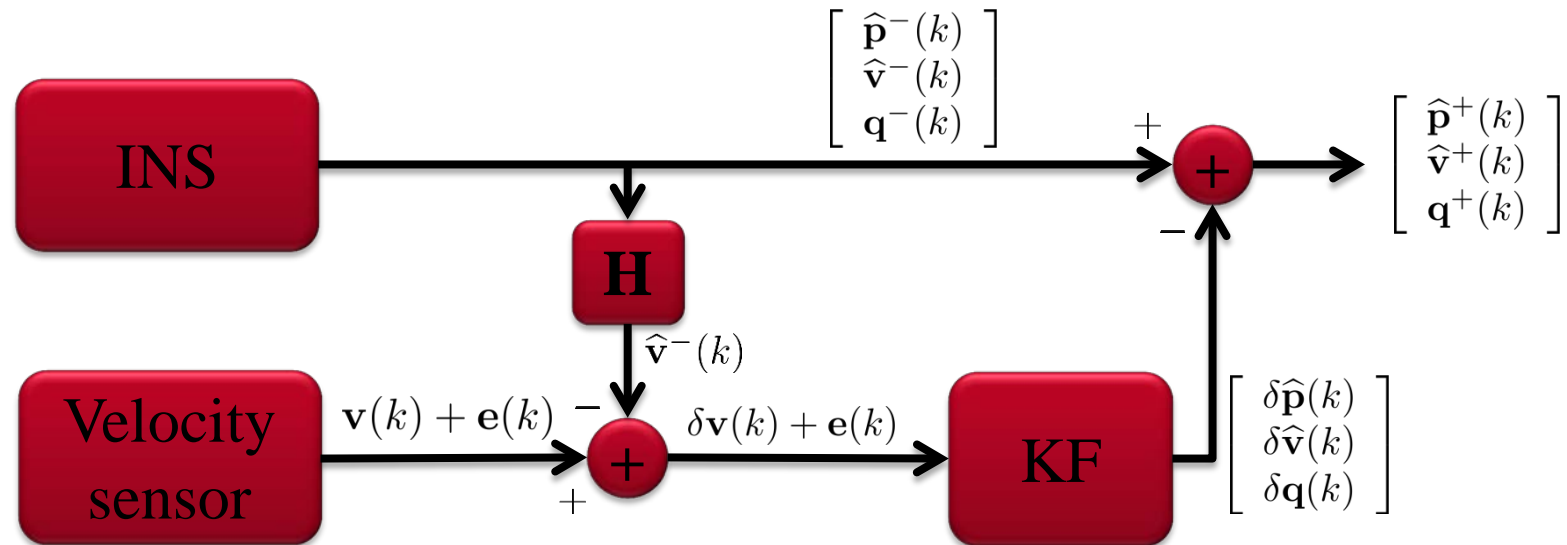
## Pros

- Simple filter structure.

## Cons

- It is difficult to model the motion dynamics in the KF framework.
- High computational load due to the high update rate of the INS.

# Complementary filtering (feed-forward)



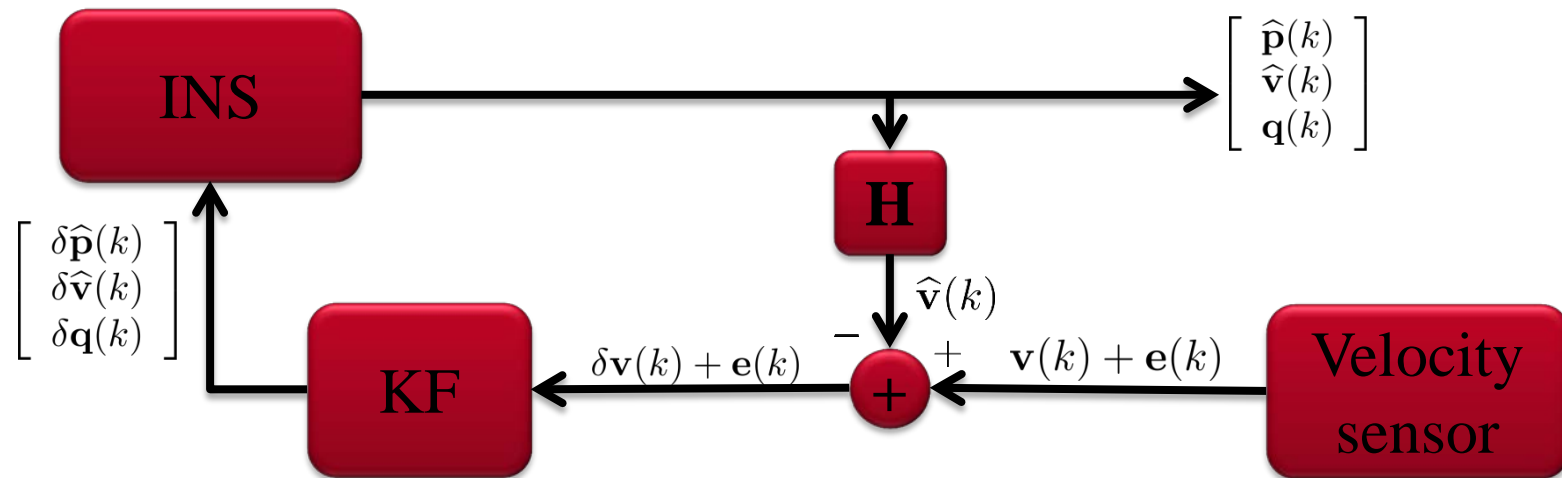
## Pros

- The KF estimates the navigation state errors, which can be better modeled in the KF framework.
- The KF needs only to be updated at the sample rate of the velocity sensor.

## Cons

- Numerical problems with low-cost INS.

# Complementary filtering (feedback)



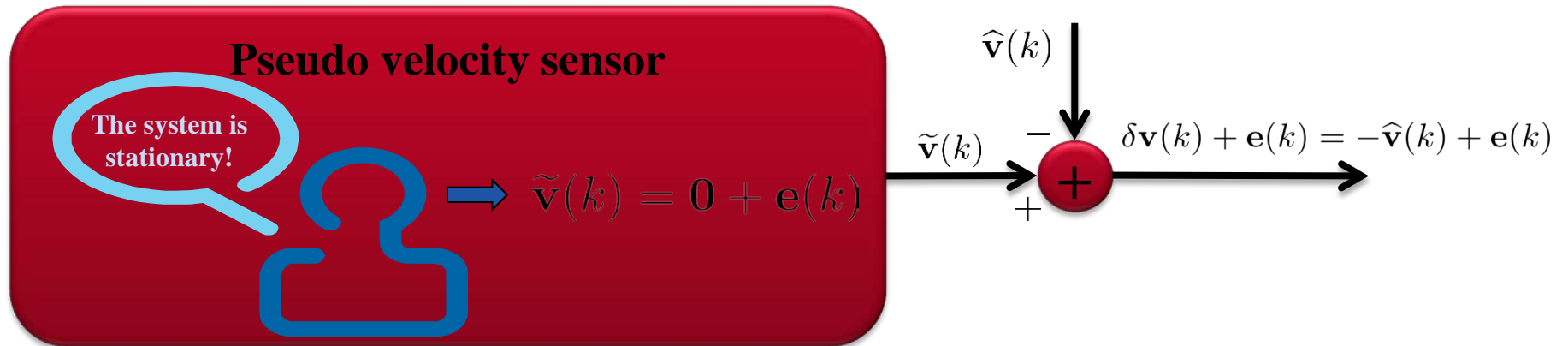
## Pros

- The KF estimates the navigation state errors, which can be better modeled in the KF framework.
- The KF needs only to be updated at the sample rate of the velocity sensor.

## Cons

- If something goes wrong in the error estimation, the navigation solution can be destroyed for all time.

# Zero-velocity updates



The observation equations for the error state space model in a zero-velocity aided INS can be written as

$$\mathbf{y}(k) = \mathbf{H}(k) \delta \mathbf{x}(k) - \mathbf{e}(k)$$

where

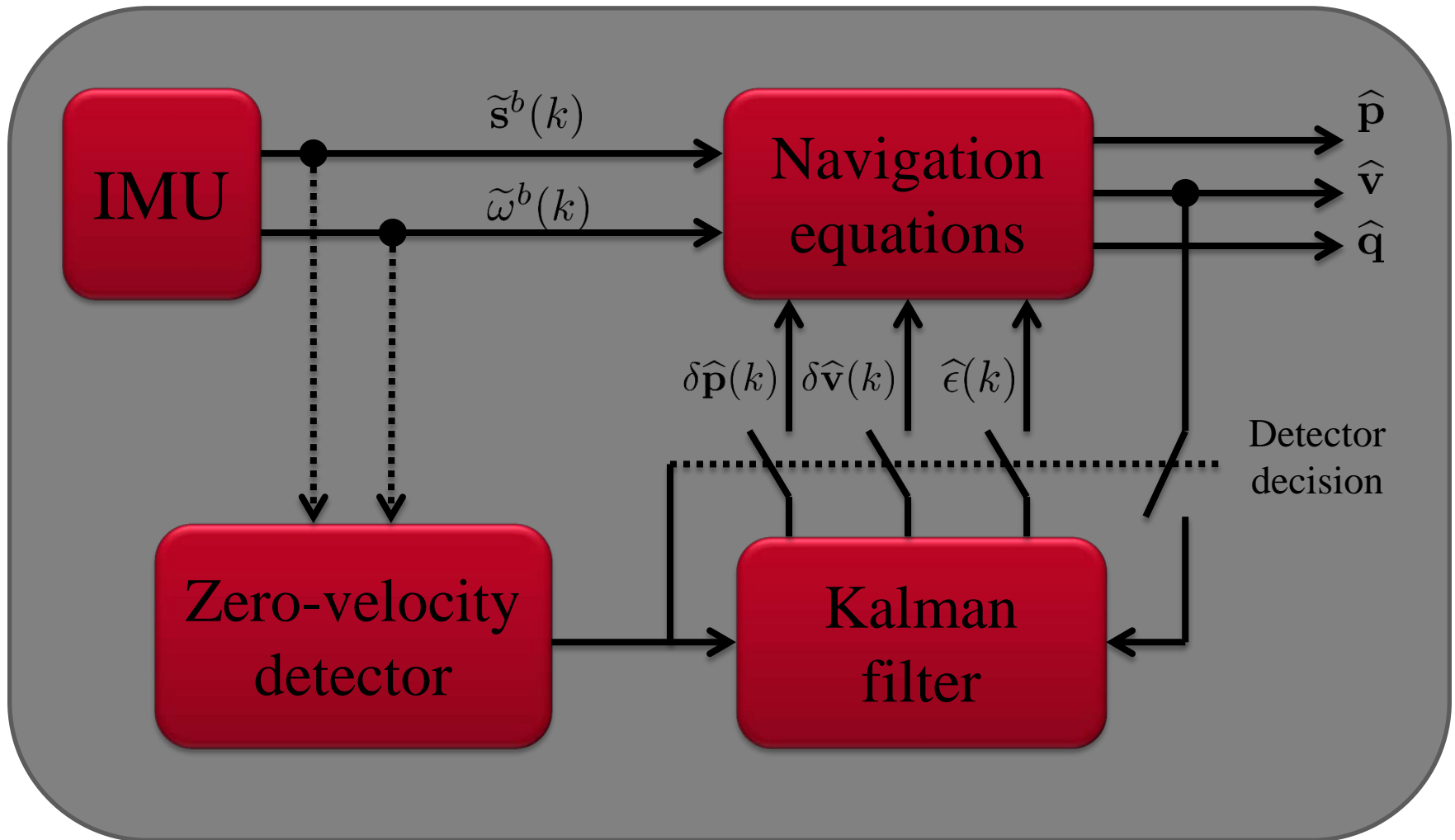
$$\mathbf{y}(k) = -\mathbf{H}(k) \hat{\mathbf{x}}(k)$$

$$\mathbf{H}(k) = \begin{cases} \mathbf{H} & \text{System is stationary} \\ \mathbf{0}_{3,9}, & \text{Otherwise} \end{cases}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{0}_{3,3} & -\mathbf{I}_3 & \mathbf{0}_{3,3} \end{bmatrix}$$

$\mathbf{e}(k)$  is additive white noise with covariance matrix  $\mathbf{R}$

# The zero-velocity aided INS Kalman filter structure.



# Pseudo code for the KF based zero-velocity aided INS

```
 $\mathbf{P}(0) \leftarrow \mathbf{Process}\{\text{Initial state covariance matrix}\}$   
 $\hat{\mathbf{x}}(0) \leftarrow \mathbf{Process}\{\text{Initial navigation state}\}$   
loop  
   $\hat{\mathbf{x}}(k) \leftarrow f(\hat{\mathbf{x}}(k-1), \tilde{\mathbf{s}}(k), \tilde{\omega}(k))$   
   $\mathbf{P}(k) \leftarrow \mathbf{F}(k) \mathbf{P}(k-1) \mathbf{F}^T(k) + \mathbf{G}(k) \mathbf{Q} \mathbf{G}^T(k)$   
   $T(k) \leftarrow \mathbf{Process}\{\text{Zero-velocity detector}\}$   
  if  $T(k) \leq \gamma$  then  
     $\mathbf{y}(k) \leftarrow -\mathbf{H}\hat{\mathbf{x}}(k)$   
     $\mathbf{K} \leftarrow \mathbf{P}(k) \mathbf{H}^T (\mathbf{H} \mathbf{P}(k) \mathbf{H}^T + \mathbf{R})^{-1}$   
     $\delta\hat{\mathbf{x}}(k) \leftarrow \mathbf{K} \mathbf{y}(k)$   
     $\hat{\mathbf{x}}(k) \leftarrow \Gamma(\hat{\mathbf{x}}(k), \delta\hat{\mathbf{x}}(k))$   
     $\mathbf{P}(k) \leftarrow (\mathbf{I} - \mathbf{K} \mathbf{H}) \mathbf{P}(k)$   
     $k \leftarrow k + 1$   
  end if  
end loop
```

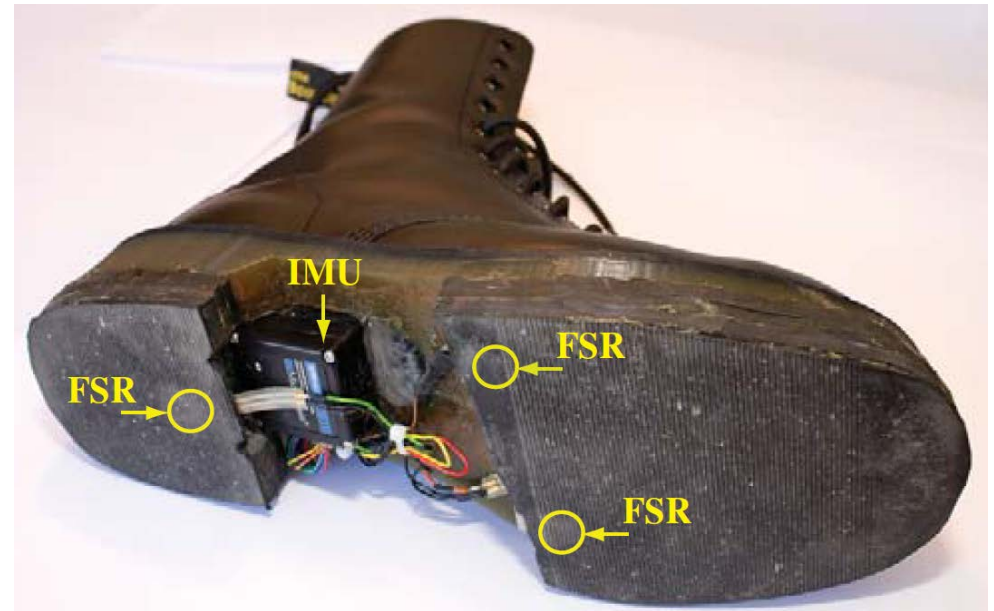
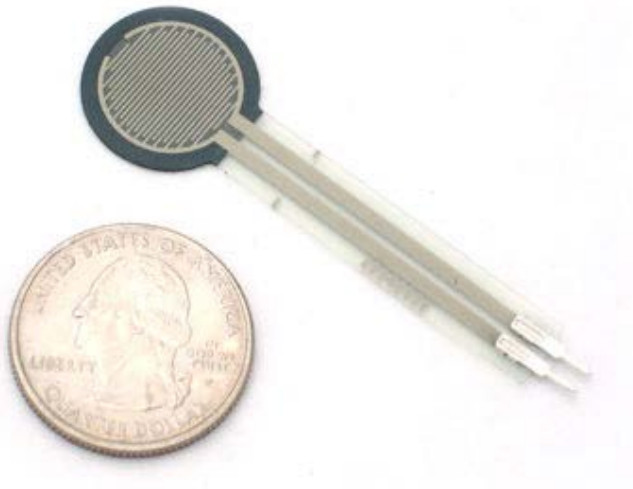


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# Zero-velocity detection

- Force sensitive resistors as a detector
- Zero-velocity detection using IMU data
- The SHOE detector

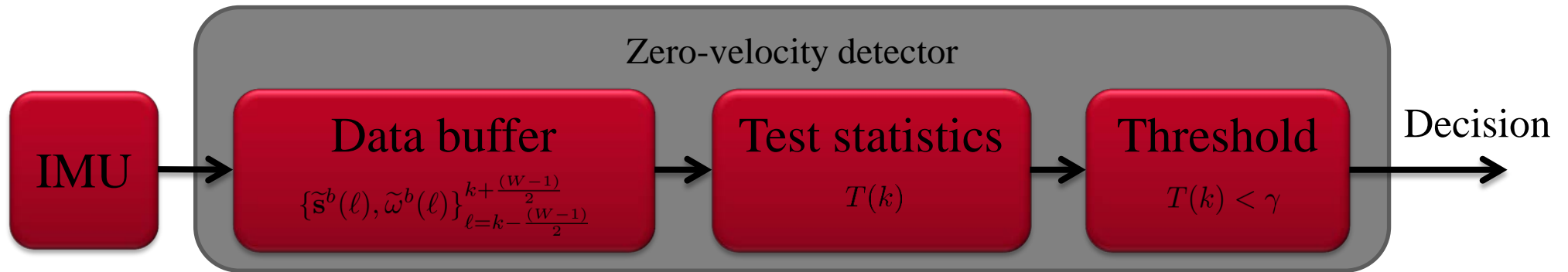
# Force sensitive resistors (FSR) as zero-velocity detector



## Drawbacks

- Sensitive to mechanical fatigue
- Threshold is weight dependent
- Only works when pressure is applied

# Zero-velocity detection using IMU data



## When the system is stationary, then

- The specific force measured by the accelerometers is equal to the gravitation acceleration, whose magnitude is known.
- The attitude of the IMU is constant, i.e., the angular rate experienced by the IMU is zero.



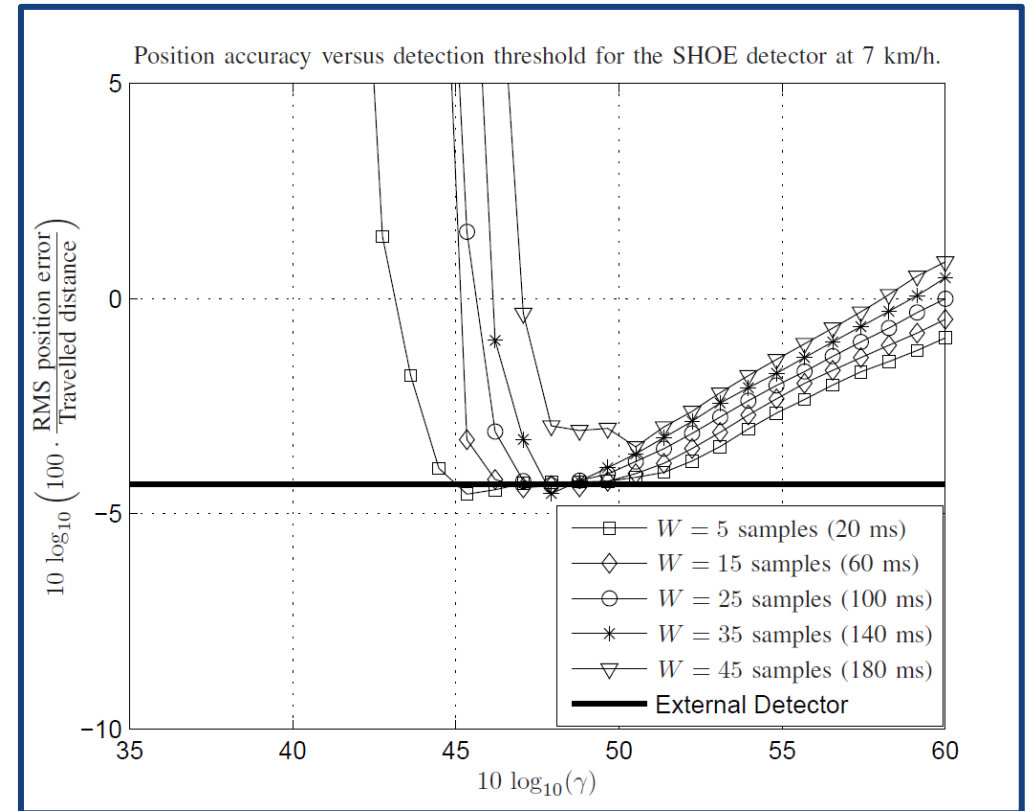
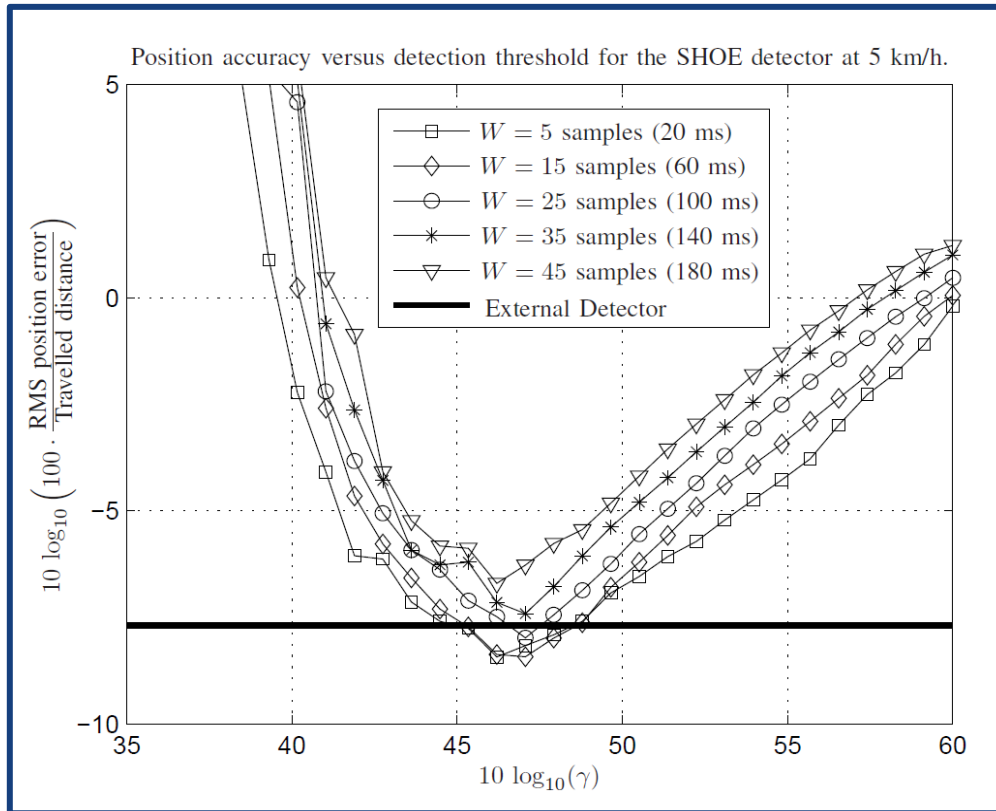
## The SHOE detector

$$T(k) = \frac{1}{W} \sum_{\ell=k-\frac{W-1}{2}}^{k-\frac{W-1}{2}} \left( \frac{1}{\sigma_a^2} \|\tilde{\mathbf{s}}^b(\ell) - g \frac{\bar{\mathbf{s}}(k)}{\|\bar{\mathbf{s}}(k)\|} \|^2 + \frac{1}{\sigma_\omega^2} \|\tilde{\boldsymbol{\omega}}(\ell)\|^2 \right)$$

where

$$\bar{\mathbf{s}}(k) = \frac{1}{W} \sum_{\ell=k-\frac{W-1}{2}}^{k-\frac{W-1}{2}} \tilde{\mathbf{s}}^b(\ell)$$

# Position error as a function of the detector settings.



## Conclusions

- SNR is high  $\rightarrow$  Keep the window size  $W$  small to get a fast detector.
- The “optimal” threshold  $\gamma$  varies little with the gait speed.



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# The OpenShoe project

- Introduction
- Hardware
- Software
- Demo

# OpenShoe – Foot-mounted INS for Every Foot



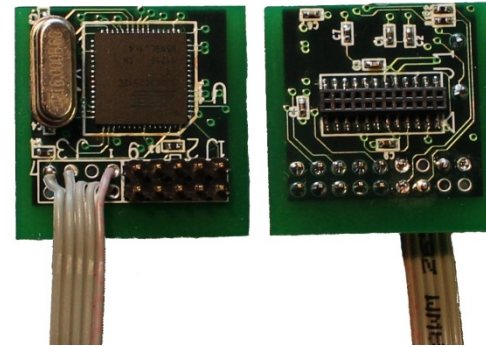
[www.openshoe.org](http://www.openshoe.org)

- OpenShoe is an open source embedded foot-mounted INS implementation including both hardware and software design.
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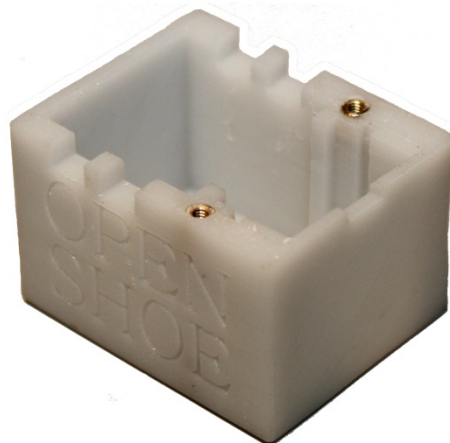
# Hardware



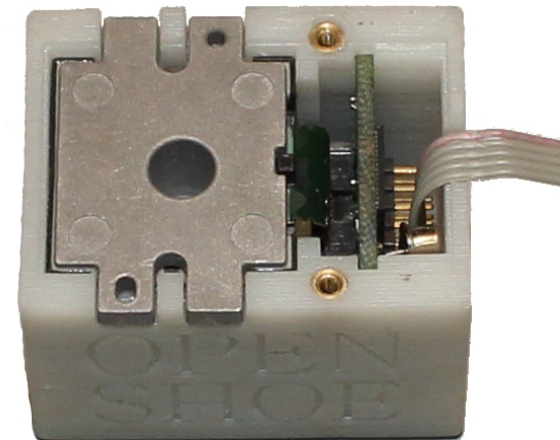
IMU



Microcontroller board



Casing



Embedded system

# Software

